1

Stabilizing Moduli Spaces of Self Dual Connections

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Let X be a smooth, closed, simply connected four dimensional manifold and let $\mathcal{M}_k^{G_n}(X)$ denote the space of based gauge equivalence classes anti - self dual connections on a principal G_n - bundle of Chern class k. For our purposes G_n will denote either $SU(n)$ or $Sp(n)$. This space can be "stabilized" by letting k tend to infinity via Taubes' procedure of gluing in one - instantons. One of the most important results in this theory is due to Taubes [0] who showed that when one takes the limit over this gluing procedure and defines

$$\mathcal{M}_\infty^{G_n}(X) = \lim_{k\to\infty} \mathcal{M}_k^{G_n}(X)$$

then one obtains a homotopy equivalence with a connected component of the entire space of gauge equivalence classes of connections,

$$\mathcal{M}_\infty^{G_n}(X) \simeq \mathcal{B}_0(X) \simeq Map_0(X, BG_n).$$

The above mapping space refers to a connected component of the space of continuous, basepoint preserving maps from X to the classifying space BG_n (all connected components in this mapping space have the same homotopy type).

The goal of this paper is to announce and outline proofs of results of joint work with M. Sanders, in which we compute the *stable homology* of the moduli space $\mathcal{M}_k^{SU(2)}(S^4)$. This is defined as follows.

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Definition 1 *The "stable homology" of a moduli space $H_*^\sigma(\mathcal{M}_k^{G_n}(X))$ is defined to be the image under the Taubes stabilzation map*

$$H_*^\sigma(\mathcal{M}_k^{G_n}(X)) = Image\left[H_*(\mathcal{M}_k^{G_n}(X)) \to \lim_{q\to\infty} H_*(\mathcal{M}_q^{G_n}(X)) = H_*(\mathcal{M}_\infty^{G_n}(X))\right].$$

Notice that by Taubes' result mentioned above, the stable homology is isomorphic to the image of the homology of the moduli space inside the homology of the space of all gauge equivalence classes of connections:

$$H_*^\sigma(\mathcal{M}_k^{G_n}(X)) = Image\left[H_*(\mathcal{M}_k^{G_n}(X) \to H_*(\mathcal{B}_k(X)) = H_*(Map_k(X, BG_n))\right].$$

Here Map_k denotes the component of the mapping space consisting of maps that pull back bundles of second Chern class $= k$.

Roughly speaking, our main result states that the stable homology

$$H_*^\sigma(\mathcal{M}_k^{SU(2)}(S^4)) \subset H_*(\mathcal{B}_k(S^4)) = H_*(\Omega^4 BSU(2)) = H_*(\Omega^3 S^3)$$

is completely detected by the image of the homology of the subspace of k - instantons that have k - distinct centers. That is, the stable homology is detected by the space of k - instantons constructed by gluing k one - instantons at k - distinct points in S^4. The homology of this space is easily calculated, as is its image in $H_*(\Omega^3 S^3)$.

Now the moduli spaces $\mathcal{M}_k^{G_n}(X)$ can also be stabilized by letting the rank n of the Lie group tend to infinity.

$$\mathcal{M}_k^{SU}(S^4) = \lim_{n\to\infty} \mathcal{M}^{SU(n)}(S^4)$$

We can then similarly define the *rank stable* homology $H_*^\rho(\mathcal{M}_k^{G_n}(X))$ as follows.

Definition 2 *The "rank stable homology" of a moduli space $H_*^\rho(\mathcal{M}_k^{G_n}(X))$ is defined to be the image*

$$H_*^\rho(\mathcal{M}_k^{G_n}(X)) = Image\left[H_*(\mathcal{M}_k^{G_n}(X)) \to \lim_{m\to\infty} H_*(\mathcal{M}_k^{G_m}(X)) = H_*(\mathcal{M}_k^{G_\infty}(X))\right].$$

Our next result is that for $X = S^4$, $G_n = Sp(n)$, and taking homology with mod 2 coefficients, then these two versions of stable homology (i.e stabilizing with respect to the Chern class, or with respect to rank) are isomorphic:

$$H_*^{\sigma}(\mathcal{M}_k^{Sp(n)}(S^4); \mathbb{Z}_2) \cong H_*^{\rho}(\mathcal{M}_k^{Sp(n)}(S^4); \mathbb{Z}_2).$$

In order to state these calculational results more precisely, we begin by defining the space of "k - glued one instantons" carefully. First define the configuration space of k - distinct points in $S^4 - \infty \cong \mathbb{R}^4$:

Definition 3 *Define the configuration space*

$$F(\mathbb{R}^4, k) = \{(x_1, \cdots, x_k) \in (\mathbb{R}^4)^k : x_i \neq x_j \text{ } if \quad i \neq j\}.$$

Notice that the symmetric group Σ_k acts freely on $F(\mathbb{R}^4, k)$ by permuting coordinates. It therefore has a free diagonal action on the product $F(\mathbb{R}^4, k) \times (\mathcal{M}_1)^k$, where, for ease of notation we are letting $\mathcal{M}_1 = \mathcal{M}_1^{SU(2)}(S^4)$. The space of "$k$ - glued one instantons" is the orbit space

$$F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k.$$

One should think of a point in this orbit space as a configuration of k distinct points in $\mathbb{R}^4 = S^4 - \infty$ that are labelled by one instantons. There is a map from the space of such configurations to $\mathcal{M}_k = \mathcal{M}_k^{SU(2)}(S^4)$ given by first gluing in the one instantons in small neighborhoods of the points to construct an "approximate k - instanton", which by Taubes theory [0] lies in a regular, "tubular" neighborhood of $\mathcal{M}_k \hookrightarrow \mathcal{B}_k$. The element of the moduli space $\mathcal{M}_k$ determined by this configuration is the image of the glued, approximate instanton under the retraction of this tubular neighborhood onto $\mathcal{M}_k$.

Alternatively, it is easy to directly construct the map

$$j_k : F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k \to \mathcal{M}_k$$

on the level of the ADHM construction (as was done in [0]) or on the level of Donaldson's monad construction of instantons [0], as was done in [0]. In particular, using the ADHM construction, [0] recall that elements of $\mathcal{M}_k$ are represented by pairs (Λ, B), where Λ is a k - dimensional quaternion vector amd B is a $k \times k$ quaternion matrix that satisfy certain nondegeneracy and reality equations. The image of $j_k : F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k \to \mathcal{M}_k$ are represented by ADHM pairs $(\Lambda,\, B)$, where all of the coordinates of Λ are nonzero and where B is a diagonal matrix with distinct diagonal entries.

The main result we are announcing in this paper is the following.

Theorem 1 *The stable homology $H^{\sigma}_*(\mathcal{M}_k)$ is given by the image in homology of the composition*

$$F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k \xrightarrow{j_k} \mathcal{M}_k \hookrightarrow \mathcal{M}_\infty \simeq \mathcal{B}_k \simeq \Omega^3_k S^3.$$

That is, the image in homology of $\mathcal{M}_k \hookrightarrow \mathcal{M}_\infty$ is equal to the image in homology of the k - glued one instantons, $F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k \to \mathcal{M}_k \hookrightarrow \mathcal{M}_\infty$.

We will also describe the proof of the following.

Theorem 2 *There is a natural isomorphism between the mod 2 stable homology and the rank stable homology,*

$$\Phi : H^{\sigma}_*(\mathcal{M}^{Sp(n)}_k(S^4); \mathbb{Z}_2) \xrightarrow{\cong} H^{\rho}_*(\mathcal{M}^{Sp(n)}_k(S^4); \mathbb{Z}_2).$$

Thus both types of stable homologies are detected by glued one instantons. The space of glued one instantons can be viewed as generalized t'Hooft instantons. This is because if, given a configuration of k distinct points in $\mathbb{R}^4$, one glues in a canonical one instanton in a neighborhood of each point the one obtains the "t'Hooft" instantons described by Atiyah and Jones in [0]. If $\iota \in \mathcal{M}_1$ is a fixed, canonical one instanton, then the t'Hooft instantons are given by the composition

$$F(\mathbb{R}^4, k)/\Sigma_k = F(\mathbb{R}^4, k) \times_{\Sigma_k} (\iota)^k \hookrightarrow F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1)^k \xrightarrow{j_k} \mathcal{M}_k.$$

In terms of the ADHM construction the t'Hooft instantons are represented by pairs $(\Lambda,\, B)$, where $\Lambda = (1, \cdots, 1)$ and B is a diagonal quaternionic matrix with distinct diagonal entries. Using the theory of iterated loop spaces, Atiyah and Jones computed the image in homology of the t'Hooft instantons in space of all connections,

$$H_*(F(\mathbb{R}^4, k)/\Sigma_k) \to H_*(\mathcal{B}_k) \cong H_*(\Omega^3 S^3).$$

This theory extends to give a calculation of the image of the space of glued one instantons

$$H_*(F(\mathbb{R}^4,k)\times_{\Sigma_k}(\mathcal{M}_1)^k)\to H_*(\mathcal{B}_k)\cong H_*(\Omega^3S^3),$$

and thus by theorem 1, it results in an explicit calculation of the stable homology, $H^\sigma_*(\mathcal{M}_k)$.

This calculation depends on the following observations due to Boyer and Mann [0]. $\mathcal{M}_1$ is homeomorphic to $D^5\times SO(3)$, and hence naturally homotopy equivalent to $SO(3)$. Also, as mentioned above, $\mathcal{B}_1\simeq\Omega^3_1S^3$. Then with respect to these homotopy equivalences, $\mathcal{M}_1\hookrightarrow\mathcal{B}_1$ is homotopic to the map $J: SO(3)\to\Omega^3_1S^3$, where $J(A): S^3\to S^3$ is the one point compactification of the rotation of $\mathbb{R}^3$ determined by $A\in SO(3)$. Moreover, with respect to these same homotopy equvalences, the map

$$F(\mathbb{R}^4,k)\times_{\Sigma_k}(\mathcal{M}_1)^k\to\mathcal{B}_k\simeq\Omega^3_kS^3$$

is homotopic to the composition

$$F(\mathbb{R}^4,k)\times_{\Sigma_k}(SO(3))^k\xrightarrow{J_k}\Omega^4_kS^4=\Omega^4_k\mathbb{HP}^1\hookrightarrow\Omega^4_k\mathbb{HP}^\infty\simeq\Omega^3_kS^3$$

where

$$J_k(x_1,\cdots,x_k;A_1,\cdots A_k): S^4\to S^4$$

is a map whose support lies in (canonically chosen) small disks around $\{x_1,\cdots,x_k\}$ and when restricted to the ith disk, the map is given by $J(A_i)$.

This is a well known construction in loop space theory, and the induced map in homology is well understood and can be described explicitly in terms of "Dyer - Lashof" homology operations (see the thesis of F. Cohen, published in [0]). These operations in the context of instantons were studied in detail by Boyer and Mann in [0]. In any case a corollary of theorem 1 is that the entire stable homology, $H^\sigma_*(\mathcal{M}_k)$ can be explicitly described in terms of these Dyer Lashof operations.

We will now give an outline of the proofs of theorems 1 and 2. If one is content with computing with mod 2 coefficients, $H^\sigma_*(\mathcal{M}_k;\mathbb{Z}_2)$, then the argument will follow rather quickly from some characteristic class calculations of the index bundle of the Dirac operator done in Sanders' thesis [0]. We begin by recalling the definition of this index bundle.

Let P_k be a principal $Sp(n)$ bundle over $S^4 = \mathbb{HP}^1$ of Chern class k, and let E_k be the associated Quaternionic vector bundle. Let A be a connection on P_k , and consider the associated Dirac operator

$$D_A : \Gamma(S^+ \otimes E_k) \to \Gamma(S^- \otimes E_k)$$

where $S = S^+ \oplus S^-$ is the canonical bundle of complex spinors on S^4. This is an elliptic operator of index k.

Since $Spin(4) = Sp(1) \times Sp(1)$, the spin bundle S has a quaternionic structure. The tensor product of the quaternion structure on S with that on E_k defines a real structure on $S \otimes E_k$. This real structure is preserved by the Dirac operator D_A, and so, upon completion, it defines a Fredholm operator on a real Hilbert space. The space of such operators is a model for the infinite classifying space BO, and therefore this construction defines a map

$$D : \mathcal{B}_k^{Sp(n)} \to BO.$$

Now by a standard Bochner type vanishing theorem, if the connection A is anti-self dual, then $coker(D_A) = 0$, and hence it has a kernel of dimension k. Associating this kernel to every point in $\mathcal{M}_k^{Sp(n)}$ defines a k dimensional vector bundle. This index bundle is classified by a map

$$D : \mathcal{M}_k^{Sp(n)} \to BO(k)$$

which makes the following diagram homotopy commute:

$$\begin{array}{ccc} \mathcal{M}_k^{Sp(n)} & \xrightarrow{D} & BO(k) \\ \iota \downarrow & & \downarrow \iota \\ \mathcal{B}_k^{Sp(n)} & \xrightarrow[D]{} & BO. \end{array}$$

Now as argued by Atiyah and Jones in [0] and modified somewhat in [0], with respect to the homotopy equivalence

$$h : \mathcal{B}_k^{Sp(n)} \xrightarrow{\simeq} \Omega^3_k Sp(n),$$

the Dirac map $D : \mathcal{B}_k^{Sp(n)} \to BO$ is homotopic to the composition

$$\beta_n : \Omega^3 Sp(n) \hookrightarrow \Omega^3 Sp \simeq \mathbb{Z} \times BO$$

restricted to the k^{th} component. The homotopy equivalence $\Omega^3 Sp \simeq \mathbb{Z} \times BO$ in this composition is given by (real) Bott periodicity. Thus we have a homotopy commutative diagram

$$\begin{array}{ccc} \mathcal{M}_k^{Sp(n)} & \xrightarrow{D} & BO(k) \\ {\scriptstyle u_k}\downarrow & & \downarrow{\scriptstyle \iota} \\ \Omega^3_k Sp(n) & \xrightarrow[\beta_n]{} & BO. \end{array}$$

where the map u_k is the composition

$$u_k : \mathcal{M}_k^{Sp(n)} \xrightarrow{\iota} \mathcal{B}_k^{Sp(n)} \underset{\simeq}{\xrightarrow{h}} \Omega^3_k Sp(n).$$

The commutativity of this diagram implies that the image in homology of the Dirac map

$$D_* : H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2)$$

is contained in the intersection of the image subspaces

$$\beta_*(H_*(\Omega^3 Sp(n)); \mathbb{Z}_2) \cap H_*(BO(k); \mathbb{Z}_2) \subset H_*(BO; \mathbb{Z}_2).$$

This intersection is computed easily using standard techniques in homotopy theory (e.g Dyer - Lashof operations). See [0] for details. In that paper the following was proved.

Proposition 3 *The image in homology of the Dirac map*

$$D_* : H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2)$$

is ***equal*** *to the intersection of the image subspaces*

$$\beta_*(H_*(\Omega^3 Sp(n)); \mathbb{Z}_2) \cap H_*(BO(k); \mathbb{Z}_2) \subset H_*(BO; \mathbb{Z}_2).$$

This proposition is proved by restricting the Dirac homomorphism D_* to the homology of the space of k - glued one - instantons, $H_*(F(\mathbb{R}^4, k) \times_{\Sigma_k}$

$(\mathcal{M}_1^{Sp(n)})^k;\mathbb{Z}_2)$. Now it is well known, for example by considering the ADHM construction, that $\mathcal{M}_1^{Sp(n)} \simeq \mathbb{RP}^{4n-1}$ and so there is an natural map

$$\nu : F(\mathbb{R}^4,k) \times_{\Sigma_k} (\mathcal{M}_1^{Sp(n)})^k \simeq F(\mathbb{R}^4,k) \times_{\Sigma_k} (\mathbb{RP}^{4n-1})^k \to E\Sigma_k \times_{\Sigma_k} (\mathbb{RP}^\infty)^k.$$

However the space $E\Sigma_k \times_{\Sigma_k} (\mathbb{RP}^\infty)^k$ is the classifying space of the wreath product group $\Sigma_k \int O(1)$, which is naturally a subgroup of $O(k)$. Hence there is an induced map on classifying spaces

$$\omega : E\Sigma_k \times_{\Sigma_k} (\mathbb{RP}^\infty)^k \to BO(k)$$

which is well known to induce a surjective map in homology with $\mathbb{Z}_2$ coefficients. This is a variant of the "splitting principle". Moreover, by studying how the Dirac operator behaves with respect to Whitney sum it can be seen that the following diagram homotopy commutes:

$$\begin{tikzcd}
F(\mathbb{R}^4,k) \times_{\Sigma_k} (\mathcal{M}_1^{Sp(n)})^k \arrow[r, "\nu"] \arrow[d, "j_k"'] & E\Sigma_k \times_{\Sigma_k} (\mathbb{RP}^\infty)^k \arrow[d, "\omega"] \\
\mathcal{M}_k^{Sp(n)} \arrow[r, "D"'] & BO(k).
\end{tikzcd}$$

Thus to compute the image of the Dirac homomorphism restricted to the homology of glued one instantons,

$$H_*(F(\mathbb{R}^4,k) \times_{\Sigma_k} (\mathcal{M}_1)^k;\mathbb{Z}_2) \xrightarrow{(j_k)_*} H_*(\mathcal{M}_k^{Sp(n)};\mathbb{Z}_2)$$
$$\xrightarrow{D_*} H_*(BO(k);\mathbb{Z}_2) \hookrightarrow H_*(BO;\mathbb{Z}_2)$$

one only has to compute the image of the composition

$$H_*(F(\mathbb{R}^4,k) \times_{\Sigma_k} (\mathbb{RP}^{4n-1})^k;\mathbb{Z}_2) \xrightarrow{\nu_*} H_*(E\Sigma_k \times_{\Sigma_k} (\mathbb{RP}^\infty)^k;\mathbb{Z}_2)$$
$$\xrightarrow{\omega_*} H_*(BO(k);\mathbb{Z}_2) \hookrightarrow H_*(BO;\mathbb{Z}_2)$$

which is a standard and easy calculation. One proves proposition 3 by showing explicitly that this image is exactly the intersection of the groups

$$\beta_*(H_*(\Omega^3 Sp(n));\mathbb{Z}_2) \cap H_*(BO(k);\mathbb{Z}_2) \subset H_*(BO;\mathbb{Z}_2).$$

See [0] for details.

Notice that this proof of proposition 3 implies that the image of $\mathbb{Z}_2$ - homology of the Dirac map,

$$D_* : H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2)$$

is equal to the image of the Dirac map, when restricted to the space of glued one - instantons

$$H_*(F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1^{Sp(n)})^k; \mathbb{Z}_2) \xrightarrow{(j_k)_*} H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{D_*} H_*(BO; \mathbb{Z}_2).$$

But on the other hand, by the commutativity of diagram proceeding the statement of proposition 3, this says that the image of the composition

$$H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{(u_k)_*} H_*(\Omega^3 Sp(n); \mathbb{Z}_2) \xrightarrow{(\beta_n)_*} H_*(BO; \mathbb{Z}_2)$$

is equal to the image when restricted to the space of glued one instantons

$$\begin{aligned} H_*(F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1^{Sp(n)})^k; \mathbb{Z}_2) &\xrightarrow{(j_k)_*} H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \\ &\xrightarrow{(u_k)_*} H_*(\Omega^3 Sp(n); \mathbb{Z}_2) \xrightarrow{(\beta_n)_*} H_*(BO; \mathbb{Z}_2). \end{aligned}$$

However it is well known that the map in mod 2 - homology,

$$(\beta_n)_* : H_*(\Omega^3 Sp(n); \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2)$$

is a monomorphism. This implies that the stable homology, $H_*^{\sigma}(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2)$, which is the image in homology of the map

$$H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{(u_k)_*} H_*(\Omega^3 Sp(n); \mathbb{Z}_2)$$

is equal to the image when restricted to the space of glued one - instantons,

$$H_*(F(\mathbb{R}^4, k) \times_{\Sigma_k} (\mathcal{M}_1^{Sp(n)})^k; \mathbb{Z}_2) \xrightarrow{(j_k)_*} H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{(u_k)_*} H_*(\Omega^3 Sp(n); \mathbb{Z}_2).$$

This is the statement of theorem 1 (with $\mathbb{Z}_2$ coefficients) in the case of $Sp(1)$.

We observe that this argument does not work with $\mathbb{Z}_p$ - coefficients for p an odd prime, since the map $(\beta_n)_*$ is not a monomorphism in homology with mod p coefficients. For this a more delicate argument is used involving the stable homotopy type of symmetric groups. We will not go into this argument here. Details will appear in [0]. We also note that for homology with rational coefficients, the statement of theorem 1 is vacuous because $\tilde{H}_*(\Omega^3_k S^3; \mathbb{Q}) = 0$.

To prove theorem 2, we use the following result proved in [0].

Theorem 4 *The Dirac maps $D : \mathcal{M}_k^{Sp(n)} \to BO(k)$ are compatible with respect to the inclusions $\mathcal{M}_k^{Sp(n)} \hookrightarrow \mathcal{M}_k^{Sp(n+1)}$ and so define a map on the stabized space*

$$D : \mathcal{M}_k^{Sp(\infty)} \to BO(k)$$

which is a homotopy equivalence.

This theorem implies that the rank stable homology $H_*^\rho(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2)$ is equal to the image in homology of the Dirac homomorphism

$$D_* : H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \to H_*(BO(k); \mathbb{Z}_2).$$

But since the inclusion $BO(k) \hookrightarrow BO$ induces a monomorphism in mod 2 homology, this means that the rank stable homology is isomorphic to the image of the composition

$$H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{D_*} H_*(BO(k); \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2).$$

But by the commutativity of the diagram before the statement of proposition 3, this is equal to the image of the composition

$$H_*(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \xrightarrow{(u_k)_*} H_*(\Omega^3 Sp(n); \mathbb{Z}_2) \xrightarrow{(\beta_n)_*} H_*(BO; \mathbb{Z}_2).$$

Now as observed earlier, the stable homology $H_*^\sigma(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2)$ is the image of the first map in this composition, $(u_k)_*$. Thus the map

$$(\beta_n)_* : H_*(\Omega^3 Sp(n); \mathbb{Z}_2) \to H_*(BO; \mathbb{Z}_2)$$

induces a map on these image subspaces, which is to say from the stable homology to the rank stable homology:

$$\Phi : H_*^\sigma(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2) \to H_*^\rho(\mathcal{M}_k^{Sp(n)}; \mathbb{Z}_2).$$

But as observed earlier the map β_n induces a monomorphism in mod 2 homology, and hence an isomorphism on these image subspaces. That is, the above homomorphism Φ is an isomorphism, which is the statement of theorem 2.

References

[1] M.F. Atiyah, V.G. Drinfeld, N.J. Hitchen, and Y.I. Manin, *Construction of instantons*, **Phys. Lett.A** vol 65 (1978), 185-187.

[2] M.F. Atiyah and J.D.S. Jones, *Topological aspects of Yang- Mills theory*, **Comm. Math. Phys.** vol. 61 (1978), 97 - 118.

[3] C. Boyer and B. Mann, *Homology operations on instantons*, **J. Diff. Geo** vol 28 (1988), 423-465.

[4] F.R. Cohen, T.J. Lada, and J.P. May, **The Homology of Iterated Loop Spaces**, Springer Verlag LNM vol 533 (1976).

[5] R.L. Cohen and M. Sanders, *The stable homology of instantons*, in preparation.

[6] S.K. Donaldson, *Instantons and geometric invariant theory*, **Comm. Math. Phys.** vol 93 (1984), 453-460.

[7] M. Sanders, *Classifying spaces and Dirac operators coupled to instantons*, Ph.D thesis, Stanford Univ. 1994.

[8] C.H. Taubes, *Self-dual connections on four-manifolds with indefinite intersection matrix*, **J. Diff. Geo.** vol.19 (1984), 517-560.

[9] C.H. Taubes, *The stable topology of self dual moduli spaces*, **J. Diff. Geo** vol. 29 (1989), 163 - 230.