Math 53- Spring 2006 - Midterm Exam II

Please circle the name of your TA:

<div align="center">

(Eleny Ionel)   (Nadya Shirokova)

Daniel Ford   Josh Genauer

Geir Helleloid   Paul Dehaye

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Circle the time your TTh **section** meets: 10:00   11:00   1:15   2:15

Your name (print):

Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your TA's name and the time that you attend the TTh section. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. You have two hours to do all the problems.

Question	Score	Maximum
1		12
2		12
3		12
4		12
5		14
6		14
7		12
8		12
Total		100

Problem 1. (12 pts) Find a particular solution of the differential equation

$$2 \cdot y'' - y' = t$$

Problem 2. (12 pts) Find the inverse Laplace's transform of the function

$$F(s) = \frac{s^2 - 5s + 4}{s \cdot (s^2 + 1)}$$

Problem 3. (12 pts) Find the Laplace transform of the function

$$g(t) = \begin{cases} 2t & \text{for } t < 3 \\ 1 & \text{for } t \geq 3 \end{cases}$$

Problem 4. (12 pts) Find the solution to the initial value problem:

$$\begin{cases} y'' + \pi^2 y = \delta(t - 1/2) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

where $\delta(t)$ is the Delta function.

Problem 5. (14 pts) Find the general solution to the system

$$\begin{cases} x_1' = 2x_1 - 3x_2 \\ x_2' = x_1 - 2x_2 \end{cases}$$

Problem 6. (14 pts) Find the general solution to

$$\mathbf{x}' = \begin{pmatrix} 2 & -4 \\ 2 & -2 \end{pmatrix} \cdot \mathbf{x}$$

Problem 7. (12 pts) Solve the system:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 5 \\ -x_1 - x_2 + x_3 + x_4 + x_5 = 3 \\ -x_1 - x_2 - x_3 + x_4 - x_5 = 1 \end{cases}$$

Problem 8. (12 pts) Consider the system with unknowns x and y

$$\begin{cases} x + y = 2 \\ x + ay = b \end{cases}$$

where a, b are some constants. For what values of a and b does the system above have
    (a) no solution   _____
    (b) a unique solution   _____
    (c) exactly two solution   _____
    (d) more than two solutions   _____
    (**Explain your answer below!**)