section 9.3

11. $T > 0, D > 0$, nodal source.
13. $T < 0, D > 0$, nodal sink.
14. $D < 0$, saddle.

section 9.6

2. $A^2 = 0$, $e^A = I + A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$

4. $A^2 = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$, $e^A = I + A + \frac{1}{2}A^2 = \begin{pmatrix} -1 & 2 & -2 \\ -1 & 5/2 & -1/2 \\ 1 & -3/2 & 3/2 \end{pmatrix}$

5. (a) If $A^2 = \alpha A$, we'll have $A^n = \alpha^{n-1} A$ for $n = 1, 2, \cdots$ and therefore $e^A = I + A + \frac{1}{2}A^2 + \cdots = I + A + \frac{1}{2}\alpha A + \frac{1}{3!}\alpha^2 A + \cdots = I + \frac{e^{\alpha t}-1}{\alpha}A$ (b) $\alpha = 3$, $e^{tA} = I + \frac{e^{3t}-1}{3}A = \begin{pmatrix} 1+\frac{e^{3t}-1}{3} & \frac{e^{3t}-1}{3} & \frac{e^{3t}-1}{3} \\ \frac{e^{3t}-1}{3} & 1+\frac{e^{3t}-1}{3} & \frac{e^{3t}-1}{3} \\ \frac{e^{3t}-1}{3} & \frac{e^{3t}-1}{3} & 1+\frac{e^{3t}-1}{3} \end{pmatrix}$

6. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, we have $A^2 = -I$, $A^3 = -A$, so $e^{tA} = I + tA + \frac{1}{2}t^2A^2 + \cdots = \cos t \cdot I + \sin t \cdot A = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$

8. Let $J = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ with $J^2 = 0$, then $A = aI + bJ$, so $e^{tA} = e^{at}e^{btJ} = e^{at}(I + btJ) = \begin{pmatrix} e^{at} & bte^{at} \\ 0 & e^{at} \end{pmatrix}$

14. Let $B = A + I$, with $B^2 = 0$. Then $e^{tA} = e^{tB-tI} = e^{-t}e^{tB} = e^{-t}(I + tB) = \begin{pmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{pmatrix}$

18. $\lambda = -1$, let $B = A + I$ with $B^2 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \end{pmatrix}$, $B^3 = 0$. Therefore, $e^{tA} = e^{-t}(I + tB + \frac{1}{2}t^2B^2) = e^{-t}\begin{pmatrix} 1+t^2/2 & -t-t^2 & t^2/2 \\ -t & 1+t & -t \\ -t-t^2/2 & 2t+t^2 & 1-t-t^2/2 \end{pmatrix}$

26. Eigenvalues are 2 and 1. The algebraic and geometric multiplicity for 2 are both 2; the algebraic and geometric multiplicity for 1 are both 1. k for 2 is 1 and k for 1 is 1. For $\lambda = 2$, $v_1 = (1, 0, 1)'$ and $v_2 = (0, 1, 0)'$; for $\lambda = 1$, $v_3 = (-1, 2, 0)'$. Their independence can be verified by the determinant consisting of the three vectors is non-zero. A fundamental set of solutions is $e^{2t}v_1$, $e^{2t}v_2$, $e^t v_3$.

36. Calculate the characteristic polynomial, $(\lambda+2)(\lambda-3)^2$. We have a fundamental set of solutions $e^{-2t}v_1$, $e^{3t}v_2$, $e^{3t}(v_3 + tv_2)$, where $v_1 = (1, 0, 0)'$, $v_2 =$

$(0, 2, 1)', v_3 = (1/5, 3, 1)'$.