Math 53- Spring 2006 - Midterm Exam I

Please circle the name of your TA:

(Eleny Ionel)	(Nadya Shirokova)
Daniel Ford	Josh Genauer
Geir Helleloid	Paul Dehaye

Circle the time your TTh **section** meets: 10:00   11:00   1:15   2:15

Your name (print):

Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your TA's name and the time that you attend the TTh section. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Each of the ten problems is worth 10 points. You have two hours to do all the problems.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Problem 1. Solve

$$\frac{dy}{dx} = \frac{x\sqrt{y-3}}{x^2+1}$$

Problem 2. Solve

$$(x+2)y' = 3x^2 - y + 1$$

Problem 3. Check that the differential equation

$$(3x^2 + y)dx + (3y^2 + x)dy = 0$$

is exact and then solve it.

Problem 4. Find all the equilibrium solutions of the following differential equation and determine which ones are stable/unstable

$$y' = \frac{1 - y^2}{2y}$$

Problem 5. Consider the following initial value problem (IVP):

$$\begin{cases} y' = \dfrac{1 - y^2}{2y} \\ y(a) = b \end{cases}$$

(a) Without explicitly solving the equation, determine the values of a and b for which one can tell for sure that the IVP has a unique solution. (**Explain carefully why!**)

(b) Next, still without explicitly solving the equation, show that the solution of the IVP with initial condition $y(1) = 2$ satisfies $y(t) > 1$ for all t in its maximal interval of existence. (**Explain carefully why!**)

Problem 6. Solve the initial value problem

$$y'' - 2y' - 3y = 0, \quad y(0) = 3, y'(0) = 1$$

Problem 7. Solve

$$y'' - 10y' + 25y = 0$$

Problem 8. Solve

$$y'' + 2y' + 2y = 0$$

Problem 9. Compute the Wronskian of the functions x and e^{2x} and determine whether these functions are linearly dependent or independent on $\mathbb{R}$.

Problem 10. Find a second order linear, homogeneous equation with constant real coefficients that has $y(t) = e^{2t} \sin t$ as a solution. What is the general solution of this equation?