

Math 215B: Homework 7  
Due Thursday, March 5, 2020

- (1) Prove that a smooth function  $f : M \rightarrow \mathbb{R}$  is Morse if and only if its differential  $df : M \rightarrow T^*M$  is transverse to the zero section. Here  $M$  is a smooth, closed manifold and  $T^*M$  is its cotangent bundle.
- (2) Let  $M \subset \mathbb{R}^L$  be a closed, smooth submanifold. For each  $v \in S^{L-1}$  let  $f_v : M \rightarrow \mathbb{R}$  be the map  $f_v(x) = \langle v, x \rangle$ . (This is essentially orthogonal projection into the line through  $v$ .) Show that the set of  $v \in S^{L-1}$  such that  $f_v$  is a Morse function is open and dense.
- (3) Let  $M \subset \mathbb{R}^L$  be a closed, smooth submanifold.. Show that the set of points  $u \in \mathbb{R}^L$  such that the map  $x \rightarrow |x - u|^2$  is a Morse function on  $M$ , is open and dense.

*Remark.* The functions described in problem (2) are called “height functions”. The functions described in problem (3) are “distance functions”. These are both very common and highly useful examples of Morse functions.