Math 215B: Homework 5

Due Thursday, February 20 , 2020

(1). A vector bundle η is said to be stably trivial if for some $k \in \mathbb{Z}$, the Whitney sum $\eta \oplus \epsilon^k$ is a trivial vector bundle, where ϵ^k denotes the standard trivial bundle of dimension k.

Let M be an n-dimensional smooth, closed manifold, and suppose that there exists an immersion

$$f : M \times \mathbb{R}^k \to R^{n+k}.$$

(a). Prove that the tangent bundle TM is stably trivial.

(b). Show that the sphere S^n has stably trivial tangent bundle for every n. (A manifold with stably trivial tangent bundle is called "stably parallelizable".)

(c). Show that the tangent bundle $TS^2 \to S^2$ is not trivial, but $TS^2 \oplus \epsilon^1$ is trivial.

(2). Suppose $P^p \hookrightarrow M^n$ and $Q^q \hookrightarrow M^n$ are smoothly embedded closed submanifolds of M^n, which we also assume is closed. Suppose further that the submanifolds intersect transversally: $P^p \pitchfork Q^q$. Let $\nu_P \to P$ be the normal bundle of P^p in M^n, and let $P^p \hookrightarrow \eta_P$ be a tubular neighborhood.

(a). Show that the restriction of ν_P to $P^p \cap Q^q$,

$$(\nu_P)_{|_{P^p \cap Q^q}} \to P^p \cap Q^q$$

is isomorphic to the normal bundle of $P^p \cap Q^q$ in Q^q.

(b). Show that the space $\eta_P \cap Q^q$ is a tubular neighborhood of $P^p \cap Q^q$ in Q^q.