Math 215B: Homework 3

Due Thursday, January 30, 2020

(1). Let $\xi \to B$ be an n-dimensional vector bundle.

(a) Define clutching functions of the nk-dimensional k-fold tensor product bundle $\otimes^k \xi \to B$ in terms of clutching functions of ξ.

(b) Define clutching functions of the k-fold exterior product bundle $\Lambda^k \xi \to B$ in terms of clutching functions of ξ.

(2) Consider the DeRham homomorphism

$$\int : \Omega^k(M) \to C^k(M; \mathbb{R})$$

for each k.

Prove that $\int$ is a map of cochain complexes. That is,

$$\int d\omega = \delta \left(\int \omega \right)$$

where $\delta : C^k(M; \mathbb{R}) \to C^{k+1}(M; \mathbb{R})$ is the singular coboundary operator. *Hint.* Use Stokes' theorem.

(3) Suppose $G \to E \xrightarrow{p} B$ is a principal bundle that has a section $\sigma : B \to E$. Show that $p : E \to B$ is a trivial principal bundle.

(4)

(b). Notice that the tensor product of two one-dimensional vector bundles ("*line bundles*") over a space B is still a one dimensional vector bundle . Show that the set of isomorphism classes of one-dimensional (real) vector bundles over B is an abelian monoid with respect to tensor product. In particular, what is the unit of this monoid?

(c). Show that in fact this abelian monoid is an abelian group. (This is often called the "Picard group".) **Hint.** Describe the clutching functions of the inverse line bundle in terms of the clutching functions of the original line bundle.