(1) For a smooth manifold $M$, define the \textit{diagonal} to be the space
\[ \Delta_M = \{(x, y) \in M \times M \mid x = y\}. \]
The smooth structure on $M$ induces a smooth structure on $\Delta_M$ via the homeomorphism
\[ M \xrightarrow{\cong} \Delta_M \quad x \mapsto (x, x). \]

(a) Show that with this smooth structure, $\Delta_M \hookrightarrow M \times M$ is a smooth embedding.
(b) Prove that there is a diffeomorphism $\Delta_TM \cong T(\Delta_M)$, i.e. the tangent bundle of the diagonal is diffeomorphic to the diagonal of the tangent bundle.
(c) Let $A \subset M$ be a submanifold, and let $f : N \to M$ be a smooth map. Prove that $f$ is transverse to $A$ if and only if the product map
\[ f \times \iota_A : N \times A \to M \times M \]
is transverse to $\Delta_M$, where $\iota_A$ is the inclusion of $A$.

(2) For a smooth function $f : M \to N$, let the \textit{graph} of $f$ be defined by
\[ \Gamma(f) = \{(x, y) \in M \times N \mid y = f(x)\}. \]

(a) Prove that the graph $\Gamma(f) \subset M \times N$ is a submanifold.
(b) Verify that the map
\[ M \to \Gamma(f), \quad x \mapsto (x, f(x)) \]
is a diffeomorphism.
(c) Consider the graph $\Gamma(D(f)) \subset TM \times TN$ of the differential $Df : TM \to TN$. Prove there is a diffeomorphism $T(\Gamma(f)) \cong \Gamma(D(f))$.

(3) Do the following exercises in Hirsch:
- p. 20 #4, #10
- p. 27 #3, #4
- p. 32 #4