MATH 120 MIDTERM
WEDNESDAY, NOVEMBER 1, 2006

Instructions: Write your name at the top of each page. Each problem is worth 10 points. Please use complete sentences. Obviously since you are under time constraints you are not expected to write at the quality of the WIM asssignment. However, you should not simply write strings of equalities without any explanation.

(1) Find all the cyclic subgroups of the dihedral group D_{12}.

(2) State Lagrange's Theorem.

(3) Let G and H be groups and suppose $\varphi : G \to H$ is a homomorphism. Prove that $\varphi(G) := \{\varphi(g) | g \in G\}$ is a subgroup of H.

(4) Suppose $H \leq G$ and let $g_1, g_2 \in G$. Prove that $g_1 H = g_2 H$ if and only if $g_2^{-1} g_1 \in H$.

(5) What properties are necessary and sufficient for a function on $G \times S \to S$ to be a group action? (here G is a group and S is a set)

(6) Consider the action of the dihedral group D_8 on the sides of a square. Label the sides with the integers $1, 2, 3, 4$. Exhibit the image of each element of D_8 in S_4 under the induced permutation representation. (Recall that if r and s are the standard generators of D_8, then r corresponds to 90° rotation and s corresponds to reflection through the x-axis).

(7) How many conjugacy classes are there in S_4?

(8) How many elements of order 5 are contained in a group of order 20?