A proposal for research on sensor networks for smart structures

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1 Introduction

How can we detect the onset of defects in complex structures by using embedded sensors? How can we use embedded sensors, for example, to detect the formation of cracks in aircraft structures? We want to locate the defects in the structure as well as to image their size and shape and we want to do this with good resolution and with as small a number of sensors as possible.

How is this to be done, with what algorithms? How can the trade-offs between resolution and algorithmic simplicity, or what is known apriori about the structure, and the number as well as the geometrical location of the sensors be assessed? We present in this proposal our ideas on these issues as well as some relevant aspects of our previous work and experience.

2 How a sensor network detects a defects in a structure

Let us suppose that the structure occupies a region D and let $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ be the location of the sensors in the structure. For the purposes of a simplified presentation of the basic mathematical issues we assume that the sensors are point transducers (emitters and receivers) of broadband ultrasound signals (up to a MHz in frequency) with flat power spectrum. This is not the case in engineering practice but it is well known how to compensate for the non-flat frequency response of the sensors. The Fourier transform of the signal received at the sensor located at $\mathbf{x}_p$ when an impulse is emitted at $\mathbf{x}_q$ is denoted by $\hat{G}_{pq}(\omega) = \hat{G}(\mathbf{x}_p, \mathbf{x}_q, \omega)$, where $\hat{G}(\mathbf{x}, \mathbf{y}, \omega)$ is the time-harmonic Green's function for the structure. This signal is **measured** so it will, in general, be a complicated functional of the acoustic (ultrasonic) properties of the structure. The **computed** Green's function will be denoted by $\hat{G}_0(\mathbf{x}, \mathbf{y}, \omega)$. It can be obtained with a finite element discretization of the structure, if available. One of the main issues in the resolution theory of defect detection and imaging is how to deal with situations where the acoustic properties of the structure are not very well known, or not known at all. This means that the computed Green's function $\hat{G}_0(\mathbf{x}, \mathbf{y}, \omega)$ may be quite different from the real one $\hat{G}(\mathbf{x}, \mathbf{y}, \omega)$, if it is known at all.

The measured responses at the transducers are significantly different when there are defects in the structure. Let $\hat{S}_{pq}(\omega)$ be the Fourier transform of the difference of the response when there is a defect from the one when there is no defect. If the size of the defect is small compared to a typical wavelength then the measured defect response can be modeled by

$$\hat{S}^M_{pq}(\omega) = \rho(\omega)\hat{G}(\mathbf{x}_p, \mathbf{y}, \omega)\hat{G}(\mathbf{y}, \mathbf{x}_q, \omega)$$

where $\mathbf{y}$ is the unknown location of the defect and $\rho(\omega)$ its scattering strength. The superscript M is used to distinguish the model from the actual measured defect response $\hat{S}_{pq}(\omega)$. The validity of such a model can be easily checked from the singular value decomposition of this measured response, where one dominant singular value emerges and all other ones are below some noise level.

If several singular values are above the noise level but their number is less than N, the size of the response matrix, then we are dealing with a collection of small defects rather than an extended one. Broadband defect detection with the singular value decomposition, and with the effect of random inhomogeneities in the medium taken into consideration, is presented in [2, 3] where many other references to the extensive literature are given.

Let $\hat{v}(\omega)$ be the singular vector of $\hat{S}_{pq}(\omega)$ associated with the top singular value $\hat{\sigma}(\omega)$. Then the **matched field** or interferometric imaging functional is defined by

$$I^(\mathbf{y}^S) = \int_B d\omega \Big| \frac{\hat{v}(\omega)}{\sqrt{\hat{\sigma}(\omega)}} \cdot \hat{g}_0(\mathbf{y}^S, \omega)\Big|^2$$

where $\mathbf{y}^S$ is a search or test point, $\hat{g}_0(\mathbf{y}^S, \omega)$ is the N-vector with components $\hat{G}_0(\mathbf{x}_p, \mathbf{y}^S, \omega)$, the computed (and assumed known) Green's function of the structure, and B is the bandwidth. This functional of the measured response difference matrix $\hat{S}_{pq}(\omega)$ has a peak when the search point $\mathbf{y}^S$ is near the defect location $\mathbf{y}$. It is used often in sonar detection and imaging with a background Green's function $\hat{G}_0$ constructed numerically from models based on topographic information. A resolution theory for the matched field functional when the sensors form an array and the background is homogeneous is given in [1]. Its connection with time reversal, interferometry and the imaging of extended defects (targets) is also given in [1].

3 Resolution theory for defect detection

The resolution theory for defect detection and imaging depends in an essential way on:

- What we know about the healthy or normal structure, that is, how well we know the acoustic Green's function $\hat{G}(\mathbf{x}, \mathbf{y}, \omega)$. We denote by $\hat{G}_0(\mathbf{x}, \mathbf{y}, \omega)$ a numerically computed version of this Green's function that is based on a discretized model, a finite element model for example
- The number and spatial deployment of the sensors

When we know the healthy structure very well and the defect is effectively surrounded by sensors, then we can achieve a resolution that is of the order of half the central wavelength. This is a theoretical limit that is easily derived theoretically and is repeatedly seen in experiments. But it is an idealization that is very far from what have or can afford in most applications.

What is not so well known and appears counter-intuitive at first is that if the healthy structure is complex, with many scatterers in it, and we do know it, then we can estimate the location of a defect with half wavelength resolution **using only two sensors** (transducers). This a form of the **super-resolution** seen in time reversal through random media [7, 10, 9, 8, 4, 1]. Time reversal means, in the present context, that we know exactly the (random) Green's function $\hat{G}(\mathbf{x}, \mathbf{y}, \omega)$ of the structure.

It is clear that there is here a complicated, and difficult to articulate and analyze, trade-off between how much we know about the medium and how many sensors we need to deploy suitably so as to achieve a desired defect image resolution. The understanding and quantification of this trade-off is the main thrust of the proposed research.

4 The proposed research

We have structured our proposed research in three parts: (a) when we know nothing about the structure that is being monitored with the sensors, (b) when we have local information about the structure, near a defect, and (c) when we have both global and local information about the structure.

4.1 Defect imaging with no prior information about the structure

In this case we can only use the arrival times of the echos from the defect at the sensors. We look at the difference response matrix $S_{pq}(t)$ in the time domain and estimate the time at which a large amplitude appears. We then estimate the location of the defect by a least squares triangulation assuming a homogeneous background.

There are at least two issues that need to be addressed in this approach. One is that waves in many structures, such as elastic plates and shells, are dispersive [6] and so arrival times are not easily estimated. The other is that the signal to noise ratio (SNR) of the traces of the difference response matrix must be large enough in order to have a reasonable arrival time estimate.

Both dispersive and SNR issues can be dealt with using iterated time reversal. This means that one or more transducers first emit a signal into the structure and then all sensors time-reverse and re-emit the signals that they receive, repeatedly, after subtracting each time the (measured) healthy response matrix $G_{pq}(t) = G(\mathbf{x}_p, \mathbf{x}_q, t)$. This iterative process produces a set of signals $u_p(t)$ that have much sharper arrival times than the ones in $S_{pq}(t)$. The least squares estimation of the defect location should now have much better resolution. One of the aims of this research is to model experimentally and numerically this estimation process, analyze mathematically its resolution, and compare the theoretical results with those measured in the experiments and with those computed numerically.

An additional issue here is the optimization of the resolution in the estimation process with time reversal relative to the number and location of the sensors.

4.2 Defect imaging with local knowledge of the structure

In many situations we do have information about the structure, a finite element model for example, but do not want to compute the full Green's function $\hat{G}_0(\mathbf{x}_p, \mathbf{y}^S, \omega)$ because this is a very large and extensive numerical computation. We can, however, calculate a local Green's function near the defect, and with good numerical resolution. One way to do this is by inserting a perfectly matched absorbing layer around the numerical domain of the selected region. It is now important to cut off the measured time traces beyond times that correspond to signals coming from outside the absorbing layer. Once this is done we can use the matched field functional for imaging the defects.

There is another functional, closely related to the matched field functional, that uses the measured data more efficiently and has better range resolution, which may well translate into requiring fewer sensors. This is an interferometric functional that uses the residual coherence of the time traces in the frequency domain. We will not describe this is detail as it is work in progress at present. Numerical simulations and some theoretical analysis indicate quite clearly that we should not use the absolute value squared of the integrand in the matched field functional because coherence information is lost. It is better to use the integrand at one frequency multiplied by its complex conjugate at another nearby frequency, and then integrate locally over the difference frequency and over the bandwidth.

In the proposed project we will study this approach in detail and compare it with carefully done experiments and numerical simulations.

4.3 Defect imaging with global and local knowledge of the structure

The idea here is to use a global version of the computed Green's function $\hat{G}_0(\mathbf{x}_p, \mathbf{y}^S, \omega)$, with low numerical resolution. After a preliminary image of the defects has been obtained using some form of the matched field functional with this Green's function, we can then get a sharper image by using a locally computed Green's function in the neighborhood of the defects.

The reason we want to do this is related to the phenomenon of super-resolution in time reversal that we mentioned above. If we have even a rough version of the global Green's function, we can locate the defects with a relatively small number of sensors. The intuition behind this expectation is that the global complexity (the local one too) of the structure acts as a surrogate for additional sensors in key locations where there is a lot of scattering. We expect, therefore, that with a few real sensors may be able to provide an acceptable estimate of defect locations. We can then operate locally at much higher numerical resolution to get a better image, with some form of matched field functionals.

We want to explore this idea of hierarchical imaging experimentally and numerically, as well as to develop a theory for it.

5 Theory, experiments and numerical simulations

Our interdisciplinary engineering-applied mathematics group has considerable expertise in imaging in complex media, which is the core problem in this project.

On the theoretical side, much of the research comes down to problems in wave propagation in random media and the associated stochastic analysis and statistical estimation theory. These are areas in which we have contributed significantly over several years and with many collaborators (former graduate students and postdoctoral visitors).

On the computational side we have very well tested codes for computational wave propagation, constructed and analyzed C. Tsogka (now a postdoctoral visitor at Stanford and next year a tenure track assistant professor in the US) who is an expert in finite element time domain methods. All the results in [2, 3] as well as in [5] are obtained with the codes she has developed in 2D and 3D and with perfectly matched absorbing layers (including codes for elastic waves). With our postdoctoral visitor G. Derveaux, who is also an expert in computational wave propagation, we are starting a series of simulations for imaging in dispersive structures, such as elastic plates.

On the experimental side ...

References

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