Optimally Designed Time Reversal and Zero Forcing Schemes

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***Abstract*— Assuming perfect channel knowledge at the transmitter, we propose optimal pre-equalization filters (spatial and temporal) that expand on the conventional Time Reversal (TR) and Zero Forcing (ZF) schemes. The filters are designed to minimize Inter Symbol Interference (ISI) and maximize the received power at the intended receiver. The different schemes are compared in terms of bit error rate and the best performance is obtained for TR scheme with additional time filtering. Moreover we show that this scheme can achieve low probability of intercept around the intended receiver.**

I. Introduction

Time reversal (TR) is a spatio-temporal matched filter: when used at the transmitter in richly scattering environments it achieves spatial focusing and temporal compression on the intended receiver. These properties were observed experimentally and studied theoretically in the context of scalar acoustic waves both in the ultrasonic and underwater regimes [1]–[3]. The feasibility of underwater communications using TR has also been experimentally demonstrated [5]. In the framework of electromagnetic waves the first experimental demonstration of time reversal was given in [4].

The properties of TR make it an attractive idea for wireless communications. The spatial focusing property of TR is interesting for Low Probability of Intercept applications, as it implies that the signal can only be decoded at the intended receiver location. The time-compression property of TR indicates that the delay spread can be reduced, and thereby Inter Symbol Interference (ISI) is also reduced. This is in practice beneficial in low signal to noise ratio (SNR) situations: as the SNR increases the performance of TR saturates due to the residual ISI and results in irreducible bit error rate (BER) [9].

Zero forcing (ZF) is a more conventional form of pre-equalization. ZF schemes completely eliminate ISI and perform very well at higher SNRs. Indeed, they suffer at lower SNR (power penalty because of ISI reduction), which is the dual effect to the noise enhancement phenomenon that would arise if ZF were applied at the receiver side.

We propose in this paper a study of a more general family of TR and ZF weighting schemes that are designed to minimize ISI and maximize the received power at the intended receiver. The performance of the proposed schemes is evaluated in terms of their bit error rate (BER) on the intended receiver and its vicinity.

II. Fundamentals of TR and ZF Systems

Let us first specify the notation used throughout the paper: Functions of time are denoted with lower case letters, upper case indicates their frequency-domain representation. Bold face indicates vectors, plain characters denote scalar quantities and overline denotes the complex conjugate of the argument.

We concentrate on the analytic representation of the signal/channel: if the bandwidth of the system is B, we translate the relevant functions to $[-B/2, B/2]$. In reality, the communication occurs around a carrier frequency f_c, which is 2.5GHz or 5GHz, depending on the system used.

We consider in this paper the operation of a downlink system with N_{TX} transmit antennas. We denote as $h(t; \mathbf{r}_{\mathrm{TX}}, \mathbf{r}_{\mathrm{RX}})$ the channel impulse response from a transmitter at location $\mathbf{r}_{\mathrm{TX}}$ to a receiver at location $\mathbf{r}_{\mathrm{RX}}$.

For our purposes, we assume that the transmitters have instantaneous and perfect knowledge of the channel impulse responses to the intended receiver, which we refer to as Channel State Information (CSI).

A. Data transmission with pre-equalization

Fig. 1 shows a layout of the pre-equalization system of interest.

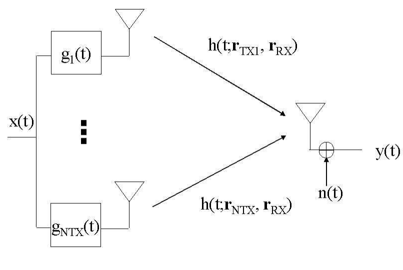


Fig. 1. Multiple transmitter pre-equalization model

Let the signal to be transmitted be $x(t)$, and let it be given by,

$$x(t) = \sqrt{P} \sum_{k=-\infty}^{\infty} \beta_k \phi(t - kT_s), \qquad (1)$$

where:

- P is the transmitted power.

- The quantity β_k denotes the mapping of the data stream b_k for the modulation scheme used. For example, for binary phase shift-keying (BPSK), bits $b_k = 0$ or $b_k = 1$ map to $\beta_k = -1$ or $\beta_k = +1$, respectively. The constellation points are normalized so that $E\left[|\beta_k|^2\right] = 1$. This (along with the normalization of $\phi(t)$ discussed later) guarantees that the transmitted power is soley determined by P.
- T_s is the symbol period. It denotes the time by which consecutive symbols β_k are separated. It is related to the system bandwidth B, namely $T_s \geq 1/B$.
- The pulse shaping function $\phi(t)$ is a low-pass function bandlimited to bandwidth B ($\Phi(f) = 0, |f| > B/2$), and we assume that $\phi(t)$ is normalized to unit power ($\int_{-\infty}^{+\infty} |\phi(t)|^2 \, dt = 1$).

  In order to simplify the derivations below, we assume that $T_s = 1/B$ and that $\phi(t)$ is the sinc function:

$$\phi(t) = \frac{1}{\sqrt{T_s}} \text{sinc}(\frac{t}{T_s}) = \frac{1}{\sqrt{T_s}} \frac{\sin(\pi \frac{t}{T_s})}{\pi \frac{t}{T_s}},$$

$$\Phi(f) = \begin{cases} \sqrt{T_s} & \text{if } |f| \leq \dfrac{B}{2} = \dfrac{1}{2T_s}, \\ 0 & \text{otherwise.} \end{cases}$$

  This choice of $\phi(t)$ is motivated by the fact that it allows for explicit and simple expression of the results that follow. Nonetheless, they can be generalized for any choice of the $\phi(t)$. The topic of more general pulse shaping functions (*e.g.* raised cosine pulses whose temporal sidelobes decay faster) is investigated in [11].

All the N_{TX} elements of the transmit array transmit simultaneously the same signal $x(t)$, by filtering it through a pre-equalization filter. In the frequency domain, the response of the m-th transmit filter $g_m(t)$ is written as $G_m(f)$. It is assumed that the filters do not introduce any signal amplification and therefore,

$$\sum_{m=1}^{\mathrm{N_{TX}}} \|g_m(t)\|^2 = \sum_{m=1}^{\mathrm{N_{TX}}} \|G_m(f)\|^2 = 1. \tag{2}$$

$\|f(x)\|^2 = \displaystyle\int |f(x)|^2 \, dx$ is the L^2 norm of the function f.

The baseband received signal $y(t)$ can be written as

$$y(t) = \sum_{m=1}^{\mathrm{N_{TX}}} g_m(t) \otimes h_m(t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}}) \otimes x(t) + n(t), \tag{3}$$

where $\otimes$ denotes the convolution operation, $h_m(t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})$ is the channel impulse response from the m-th element of the transmit array to the intended receiver at location $\mathbf{r}_{\mathrm{RX}}$, and $n(t)$ is the receiver thermal noise, which is assumed to be zero-mean Gaussian with variance σ^2.

The equivalent channel impulse response at location $\mathbf{r}_{\mathrm{RX}}$ $h_{\text{eq}}(t; \mathbf{r}_{\mathrm{RX}})$ can be written as

$$h_{\text{eq}}(t; \mathbf{r}_{\mathrm{RX}}) = \sum_{m=1}^{\mathrm{N_{TX}}} h_m(t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}}) \otimes g_m(t),$$

$$H_{\text{eq}}(f; \mathbf{r}_{\mathrm{RX}}) = \sum_{m=1}^{\mathrm{N_{TX}}} H_m(f; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}}) \cdot G_m(f).$$

The receiver samples the signal $y(t)$ at integer multiples of the symbol time T_s. Let us assume that the receiver does not perform any equalization, and tries to decode the transmitted signal based on each individual sample of $y(t)$. This decision is impaired by the noise $n(t)$, and inter-symbol interference (ISI) caused by the spread of the equivalent channel impulse response h_{eq}.

In order for the system to be ISI free, then the Nyquist criterion should be satisfied. Namely:

$$\sum_{k=-\infty}^{+\infty} H_{\text{eq}}(f - k\frac{1}{T_s}) \Phi(f - k\frac{1}{T_s}) = \text{const.} \tag{4}$$

Given our choice of $\phi(t)$, H_{eq} should satisfy

$$\sum_{k=-\infty}^{+\infty} H_{\text{eq}}(f - k\frac{1}{T_s}) = \text{const.} \tag{5}$$

B. Time Reversal systems

In the case of TR transmission, the filter $g(t)$ is the time reversed and phase conjugated version of the channel impulse response to the intended receiver.

If $g_m^{\text{TR}}(t)$ is the filter at the m-th transmit antenna, then

$$g_m^{\text{TR}}(t) = \gamma_{\text{TR}} \overline{h(-t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})},$$

where $\mathbf{r}_{\mathrm{TX_m}}$ is the location of the m-th transmit antenna and γ_{TR} is a scaling factor so that the time-reversal filters satisfy the power constraint in (2). We determine γ_{TR} in the next section.

The equivalent channel impulse response $h_{\text{eq}}^{\text{TR}}(t; \mathbf{r}_{\mathrm{RX}})$ can be written as

$$h_{\text{eq}}^{\text{TR}}(t; \mathbf{r}_{\mathrm{RX}}) = \gamma_{\text{TR}} \sum_{m=1}^{\mathrm{N_{TX}}} h(t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}}) \otimes \overline{h(-t; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})},$$

and in the frequency domain, the equivalent channel transfer function $H_{\text{eq}}^{\text{TR}}(f; \mathbf{r}_{\mathrm{RX}})$ is,

$$H_{\text{eq}}^{\text{TR}}(f; \mathbf{r}_{\mathrm{RX}}) = \gamma_{\text{TR}} \sum_{m=1}^{\mathrm{N_{TX}}} |H_m(f; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})|^2.$$

By the properties of TR, the signal $y(t)$ is expected to focus spatially at the receiver and compress temporally. However, H_{eq} does not necessarily satisfy the Nyquist criterion, and therefore there exists ISI.

C. Zero Forcing Systems

In the case of ZF transmission, the filters $g_m(t)$ are designed so as to cancel the signal spreading caused by the channel. If $g_m^{\text{ZF}}(t)$ is the filter at the m-th transmit antenna, and $G_m^{\text{ZF}}(f)$ is its frequency-domain representation then

$$G_m^{\text{ZF}}(f) = \gamma_{\text{ZF}} \frac{1}{H(f; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})} = \gamma_{\text{ZF}} \frac{\overline{H(f; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})}}{|H(f; \mathbf{r}_{\mathrm{TX_m}}, \mathbf{r}_{\mathrm{RX}})|^2},$$

where γ_{ZF} is a scaling factor so that the power constraint (2) is satisfied, and is explicitly considered in the following section.

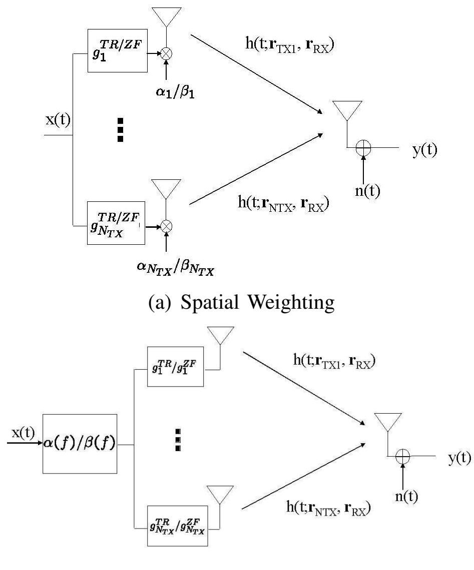
The equivalent channel impulse response $h_{\text{eq}}^{\text{ZF}}(t;\mathbf{r}_{\text{RX}})$ is: $h_{\text{eq}}^{\text{ZF}}(t;\mathbf{r}_{\text{RX}}) = \gamma_{\text{ZF}} \sum_{m=1}^{\text{N}_{\text{TX}}} h(t;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}}) \otimes g_m^{\text{ZF}}(t)$.

In the frequency domain, the equivalent channel transfer function is $H_{\text{eq}}^{\text{ZF}}(f;\mathbf{r}_{\text{RX}})$ is $H_{\text{eq}}^{\text{ZF}}(f;\mathbf{r}_{\text{RX}}) = \gamma_{\text{ZF}}\text{N}_{\text{TX}}$.

The equivalent channel impulse response is constant over the bandwidth of interest, and therefore the received signal should be ISI free.

III. Communication Schemes Description

We assume that the transmitter has perfect channel knowledge. The receiver samples the received signal and, without any advanced processing or equalization, tries to determine the signal that had been transmitted. We expand on the TR and ZF schemes introduced in Section II by allowing additional filtering at the transmitter. The filtering can be in *space* (weighting of the signal from each transmit antenna), or *frequency* (additional frequency selective filter), as shown in Fig. 2(a) and (b) respectively.



(a) Spatial Weighting

(b) Frequency Weighting

Fig. 2. Illustration of the spatial and frequency weighting schemes

The notation TR_S and TR_F stands for space and frequency dependent TR filter coefficients (similarly for the ZF schemes). We do not consider the joint optimization of spatial and spectral filtering, because of the complexity and non-convexity of the complete problem. We show that the design of the filters in space and frequency separately yields simple expressions and sufficiently good results.

The filter coefficients α_m, $\alpha(f)$, β_m and $\beta(f)$ are determined by the following criteria:

1) eliminate intersymbol interference (ISI) at the indended receiver, and/or
2) maximize the received power at the sampling instant, *i.e.* at $t = 0$. This is given by $\left|\int_{-\infty}^{+\infty} H_{eq}(f)df\right|^2$

All filters are normalized so that they do not amplify the signal, *i.e.*, their total power is unity as in (2).

A. Time Reversal Filters

1) Spatial Weighting:

$$G_m^{\text{TR}_\text{S}}(f) = \alpha_m \overline{H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}})}. \tag{6}$$

No choice of α_m can guarantee an ISI free channel. We select α_m so as to satisfy the power constraint of the filters

$$\alpha_m = \frac{1}{\sqrt{\sum_{m=1}^{\text{N}_{\text{TX}}} \|H_m(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}}))\|^2}}. \tag{7}$$

This choice of the weights corresponds to the pure time reversal scheme introduced in [8]. It can be shown that it also corresponds to the weight combination that maximizes the peak received power on the intended receiver.

2) Frequency weighting:

$$G_m^{\text{TR}_\text{F}}(f) = \alpha(f)\overline{H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}})}. \tag{8}$$

From the condition (5) and the power constraint, we obtain

$$\alpha(f) = C_{\text{TR}_\text{F}} \frac{1}{\sum_{m=1}^{\text{N}_{\text{TX}}} |H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}}))|^2}, \tag{9}$$

$$C_{\text{TR}_\text{F}} = \frac{1}{\sqrt{\int_{-B/2}^{B/2} \frac{df}{\sum_m |H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}}))|^2}}}. \tag{10}$$

B. Zero-Forcing Type Schemes

1) Spatial Weighting:

$$G_m^{\text{ZF}_\text{S}}(f) = \beta_m \left(H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}})\right)^{-1}. \tag{11}$$

By construction, the equivalent channel impulse response is ISI free. The objective of maximizing the received power (which by definition is the power at $t = 0$) is achieved by setting the weights β_m to

$$\beta_m = C_{\text{ZF}_\text{S}} \frac{1}{\int_{-B/2}^{B/2} \frac{1}{|H(f;\mathbf{r}_{\text{TX}_\text{m}},\mathbf{r}_{\text{RX}})|^2} df}, \tag{12}$$

$$C_{\text{ZF}_\text{S}} = \frac{1}{\sqrt{\sum_l \frac{1}{\int_{-B/2}^{B/2} \frac{df}{|H(f;\mathbf{r}_{\text{TX}_\text{l}}\mathbf{r}_{\text{RX}})|^2}}}}. \tag{13}$$

2) Frequency weighting:

$$G_m^{\mathrm{ZF_F}}(f) = \beta(f)\left(H(f;\mathbf{r}_{\mathrm{TX_m}},\mathbf{r}_{\mathrm{RX}})\right)^{-1}. \quad (14)$$

The equivalent channel impulse response is ISI free by construction. Maximizing the received power under the power costraint leads to

$$\beta(f) = \text{const.} = \frac{1}{\sum_l \int_{-B/2}^{B/2} \frac{df}{|H(f;\mathbf{r}_{\mathrm{TX_l}},\mathbf{r}_{\mathrm{RX}}))|^2}}. \quad (15)$$

It can be shown analytically that in terms of peak received power, the schemes are ordered as [11]

$$P_{\mathrm{TR_S}} \geq P_{\mathrm{TR_F}} \geq P_{\mathrm{ZF_S}} \geq P_{\mathrm{ZF_F}}. \quad (16)$$

IV. Channel Model

The simulations are based on the 802.11n channel model [10]. For the channel impulse response (CIR) a tap delay line model is assumed with L taps of the form $h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l)$, where h_l is the complex amplitude of the l-th tap arriving at delay τ_l. The tap amplitudes are Rayleigh distributed and follow a known power delay profile.

In terms of channel correlation, the 802.11n channel model assumes the Kronecker property, *i.e.*, the correlation on the transmitting and the receiving sides are separable, and depend on the Power Azimuth Spectrum (PAS) around the transmitter/ receiver (distribution of the angles of departure and of arrival (AoD/ AoA)).

TABLE I
Angular parameters for Model E

Cluster	1	2	3	4
Mean AoA	163.7^o	251.8^o	80^o	182^o
AS (Rx)	35.8^o	41.6^o	37.4^o	40.3^o
Mean AoD	105.6^o	293.1^o	61.9^o	275.7^o
AS (Tx)	36.1^o	42.5^o	38^o	38.7^o

In [10] 6 channel models are outlined. For our simulations, we use Model E that describes a large open space environment. The delay spread is about $100ns$, and there are four distinct temporal/ angular clusters. Fig. 3 shows the tap delay line model for this scenario (not normalized to unit total power). The cluster angular parameters are summarized in Table I. In our simulations there is no transmit signal correlation.

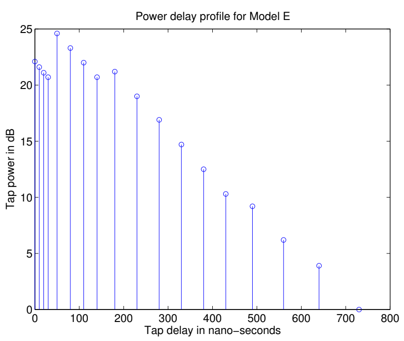


Fig. 3. Tap delay line for Model E

V. Simulation Results and Scheme Evaluation

A. Bit error rate performance

To compare the performance of the different schemes introduced in section III, we look at their BER performance on the intended receiver as a function of the SNR for the original channel (*i.e.*, without pre-filtering). BPSK modulation is used for the transmitted signal.

[7] showed that ISI can limit the achievable BER performance of a TR system in an underwater environment. This issue is accentuated in a wireless environment, due to the specifics of the radio channel and the limited number of available transmitters. At high SNR, the residual ISI of the pure TR scheme (*i.e.* $\mathrm{TR_S}$) results in irreducible BER and saturates the system performance. This suggests that, in contrast to the expectations from ultrasound and underwater sound, TR for wireless communications requires more advanced schemes. Pure TR is outperformed by the three other schemes that have zero ISI on the intended receiver. For low SNR, the performance of the schemes is determined by the delivered power at the receiver: $\mathrm{TR_S}$ is the best, then follow $\mathrm{TR_F}$, $\mathrm{ZF_S}$ and $\mathrm{ZF_F}$. However, they all have very poor BER performance.

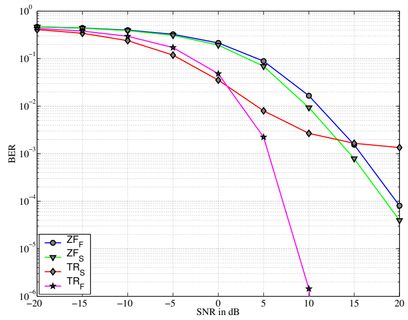
Fig. 4(a) shows that the best performance is obtained with the $\mathrm{TR_F}$ scheme which is clearly superior to the ZF schemes. The performance of the ZF schemes is more sensitive to channel fading. The $\mathrm{TR_F}$ scheme overcomes this issue by performing an inversion only after the responses from the individual elements have been summed. Due to this diversity effect, the composite channel exhibits milder fading, and therefore outperforms the ZF schemes performed on the links individually.

B. Spatial Focusing

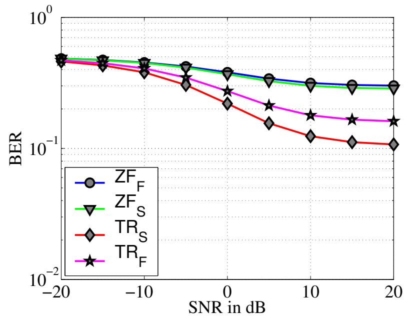
Let us assume that the system performs TR with a view to communicating with an intended receiver that is located at $\mathbf{r}_{\mathrm{RX}}$. We are interested in the probability of successful intercept of this signal at a location at distance $\mathbf{d}$ away from the intended receiver, *i.e.*, at a location $\mathbf{r}' = \mathbf{r}_{\mathrm{RX}} + \mathbf{d}$. The equivalent channel impulse response at $\mathbf{r}'$ is $h_{\mathrm{eq}}(t;\mathbf{r}') = \sum_{m=1}^{\mathrm{N_{TX}}} h(t;\mathbf{r}_{\mathrm{TX_m}},\mathbf{r}') \otimes g_m(t)$. Ideally, the BER at this off-target location is as close to 0.5 as possible.

Fig. 4(b) shows the BER at distance $\mathbf{d} = (-0.4\lambda, -0.4\lambda)$ from the intended receiver as a function of the SNR on the target. The BER is consistently above 0.1: an eavesdropper at that location would not be able to successfully intercept the content of the communication.

We now look at the average received power and average BER at locations within an area of size $4\lambda \times 4\lambda$ around the intended receiver. In Fig. 5 we plot the average maximum received power (normalized for all subplots by the power on the target for TR_s). Clearly, TR_s achieves the highest received power on the target. The relative spatial focusing is also optimal. In Fig. 6 we plot the logarithm of the BER for



(a) BER on the target



(b) BER at distance $\mathbf{d} = (-0.4\lambda, -0.4\lambda)$ from the intended receiver

Fig. 4. BER on and away from the target

SNR=15dB. The benefit of higher received power using TR_s is counteracted by the high ISI that this scheme suffers from. Clearly, TR_f is optimal in both the low BER and the high spatial focusing sense. The ZF schemes also demonstrate very good spatial focusing properties.

VI. Conclusions

We investigated pre-equalization schemes that exploit the benefits of the conventional TR and ZF techniques. We showed that pure TR is impaired by irreducible BER at high SNRs. TR augmented by an additional spectral filter can achieve low BER performance on the intended receiver, and sufficiently low probability of intercept at locations away from the intended receiver. We also investigated ZF schemes that suffer a significant power penalty, and cannot fully exploit the diversity benefit of the multiple transmitters.

Acknowldegement

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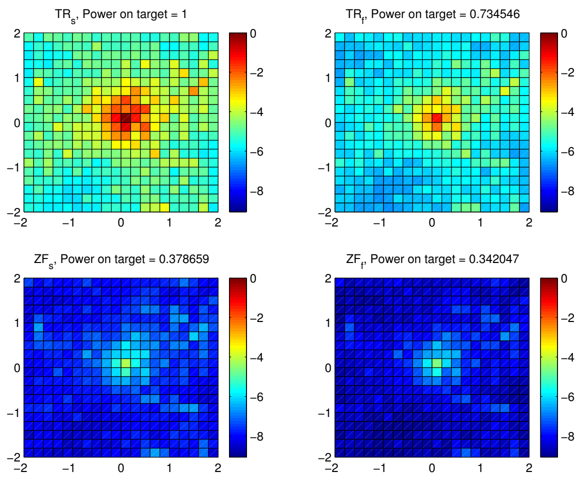

Fig. 5. Maximum received power on and around the intended receiver

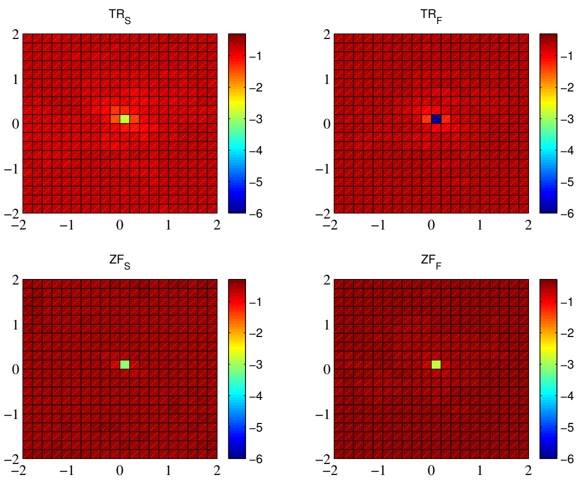

Fig. 6. BER on and around the intended receiver for SNR=15dB

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