Application of Time-Reversal with MMSE Equalizer to UWB Communications

Thomas Strohmer[1], *Majid Emami*[2], *Jan Hansen*[2], *George Papanicolaou*[3], *and Arogyaswami J. Paulraj*[2]
[1] Department of Mathematics, University of California, Davis, CA 95616-8633, USA.
Email: strohmer@math.ucdavis.edu
[2] Information Systems Laboratory, Stanford University, Stanford, CA 94305, USA.
Tel: 650-723-4102, Fax: 650-723-8473, Email: {jchansen, memami, apaulraj}@stanford.edu
[3] Department of Mathematics, Stanford University
Email:papanico@math.stanford.edu

***Abstract*— We propose to apply a technique called time-reversal to UWB communications. In time-reversal a signal is precoded such that it focuses both in time and in space at a particular receiver. Spatial focusing reduces interference to other co-existing systems. Due to temporal focusing, the received power is concentrated within a few taps and the task of equalizer design becomes much simpler than without focusing. Furthermore, temporal focusing allows a large increase in transmission rate compared to schemes that let the impulse response ring out before the next symbol is sent. Our paper introduces time-reversal, investigates the benefit of temporal focusing, and examines the performance of an MMSE-TR equalizer in an UWB channel.**

I. Introduction

Ultra-wideband (UWB) has become a suitable candidate for high-data rate, short range communications. Due to the large operation bandwidth, the resolution in delay domain is extraordinary, so that even in a dense-scattering environment, the probability of fading is low. Since for the generation of these ultra-short pulses, no filters are required, it was considered that UWB devices are cheap to produce.

Recently, however, several drawbacks have been noted. Even though UWB transmitters irradiate very low power per bandwidth, they can potentially interfere with many systems that have frequencies assigned in the low GHz range [1]. Furthermore, multipath fading may be low, but in order to catch about half of the energy that is distributed in the entire impulse response, RAKE receivers with at least 20, but potentially many more taps must be constructed [2], [3]; for handsets, such a design is not low-cost.

We claim that both these drawbacks can be overcome if UWB is combined with a transmission scheme which is called time reversal (TR). This scheme has its origin in wide-band transmission in under-water acoustics [4], [5], [6], [7] and ultra-sound [8], [7] and has recently attracted attention of wireless communications engineers [9]. In TR, the time-reversed channel impulse response (CIR) of any transmit-receive link is taken as a prefilter at the transmitter. If such a time-reversed sequence is irradiated into the channel, its components retrace their former paths and lead to a focus of power at the intended receiver at some particular time instant. For UWB communications, this strategy has several advantages. Focusing power means that the receiver needs only very few taps to capture a significant amount of the power in the channel. Essentially, the complex taks of estimating a large number of channel taps is removed from the receiver, which may be a low-cost handset, to the transmitter. In addition, the rate of the system can be increased. Whereas with proposed pulse-position modulation schemes the time between the repetition of two pulses is chosen to be sufficiently larger than the length of the CIR [10], focusing of power means that the repetition rate can be increased. At last, the spatial focusing that comes along with TR yields lower interference with other communication systems.

In this paper, we apply TR to a single user downlink scenario of an UWB channel. In order to assess the usefulness of TR, we define a time-compression factor which measures the fraction of the energy in the non-focused components of the channel versus that in the main, focused peak. For a channel with iid complex zero mean circularly symmetric taps, 50% percent of the entire CIR's energy can be captured with a single tap receiver. In real channels, this ratio is slightly lower, which we show with the aid of experimental data from UWB measurements.

Since the scattered components are usually widely distributed over the delay axis, a receiver that wants to capture more than 50% of the signal energy in the effective CIR must use some equalization technique. We propose an TR-MMSE equalizer for this purpose. We investigate the performance of this equalizer on real channel data and show that it can operate at sampling rates which are much higher than the inverse of the length of the CIR of the channel; no ringing out of the impulse response is required.

This paper is organized as follows: in the second section, we introduce TR and define and investigate the time compression factor. Also, the signal model is described and the principles of the TR-MMSE equalizer are outlined. In the third section, we first investigate the time compression factor with the aid of real data. Then, we examine the performance of the TR-MMSE equalizer in term of bit-error-rates (BER). In particular, we vary the number of taps and the sampling rate. We conclude in Section IV.

II. Theory

A. Time Reversal

The transmitter uses the time reversed complex conjugate of the CIR as the transmit prefilter. We denote this CIR by $h(\mathbf{r_0}, \tau)$, where $\mathbf{r}_0$ is the receiver location and τ is the delay variable. Applying the complex conjugate $h^*(\mathbf{r_0}, -\tau)$ as the prefilter, the effective channel to any location $\mathbf{r}$ is thus given by the time reversed field

$$g(\mathbf{r}, \tau) \triangleq \frac{1}{\sqrt{E_H}} h^*(\mathbf{r}_0, -\tau) \otimes h(\mathbf{r}, \tau) \tag{1}$$

where $\otimes$ denotes convolution and the factor $\frac{1}{\sqrt{E_H}}$ normalizes the transmit power with the square root of the channel's energy, $E_H = \int |h(\mathbf{r}_0, \tau)|^2 \mathrm{d}\tau$.

A convolution with a time-reversed signal is equivalent to a correlation. We see from (1) that the focusing relies on the decorrelation of CIRs in the delay and the spatial domain. In a rich scattering environment, a CIR can be considered as a random code sequence that is assigned to any transmit-receive pair. The auto- and crosscorrelation properties of this sequence are given by nature; unlike in CDMA, they come without any additional bandwidth spreading. In UWB, these sequences are particularly long. The benefits of time compression are crucial, and the shortening of the CIR reduces the complexity of equalization at the receiver.

The decorrelation of the CIRs is obviously best in a rich scattering environment. Physically, one can consider the ubiquitous locations of the scatterers to form a large virtual aperture which enables focusing even with a single antenna down to the diffraction limit. A simple but effective measure for the focusing capability of a TR channel is the delay spread-bandwidth product which roughly gives the number of taps that the CIRs has. For an initial evaluation of the benefits of TR we refer to [9].

B. Temporal Focusing

In a multipath environment, the channel's energy E_H is spread over delay. It can be decomposed into a term E_0 which describes the fraction of the energy in the direct path, and a term E_S, describing that of the multipaths. In a strong multipath environment such as given for UWB, we have the ratio $E_S/E_0 \gg 1$. If the channel is slowly time-variant, the quantities E_S, E_0, and E_H are random variables. Their mean is denoted by $\langle . \rangle$.

In TR, the CIR is compressed and a temporal focus of energy is visible in the center of the compressed CIR. In order to characterize the amount of the temporal focusing, we define the temporal compression ratio as

$$\gamma_{TR} = \langle E_S^{TR} \rangle / \langle E_0^{TR} \rangle = \frac{\langle E_H^{TR} \rangle}{\langle E_0^{TR} \rangle} - 1 \tag{2}$$

where E_0^{TR} is the energy in the main peak of the received impulse response, E_S^{TR} the one in the tails, and E_H^{TR} is the sum of the two.

We can compute γ_{TR} in terms of either the CIR $h(\tau)$ or the transfer function $H(\omega)$. From (1) we see that the transfer function $H^{TR}(\omega)$ of the time-reversed channel is the squared absolute value of that of $H(\omega)$. The discrete CIR $h^{TR}[l]$, with a total of L non-zero taps, is

$$h^{TR}[l] = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(\omega)|^2 \exp(j\omega l) \mathrm{d}\omega. \tag{3}$$

From this we compute the energy of the 0th tap of the time-reversed channel,

$$\begin{aligned} \langle E_0^{TR} \rangle &= \langle |h^{TR}[0]|^2 \rangle = \langle | \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(\omega)|^2 \mathrm{d}\omega |^2 \rangle \\ &= \langle (E_S + E_0)^2 \rangle = \langle E_H^2 \rangle \end{aligned} \tag{4}$$

and the energy of the entire channel,

$$\langle E_H^{TR} \rangle = \langle \sum_l |h[l] \otimes h^*[-l]|^2 \rangle = \frac{1}{2\pi} \langle \int_{-\pi}^{\pi} |H(\omega)|^4 \mathrm{d}\omega \rangle \tag{5}$$

where the frequency domain representation on the right hand side of the equation follows from Parseval's theorem. Whether a time domain or a frequency domain representation is more desirable depends on in which domain information about the channel is available.

If we evaluate expression (5) further in time domain for L identical and independently distributed taps with equal power, we can, derive the time compression factor as

$$\gamma^{TR} = \frac{\beta_4 + 2(L-1)\beta_2^2}{\beta_4 + (L-1)\beta_2^2} - 1. \tag{6}$$

Here, β_2 and β_4 denote the second and the fourth moment of the amplitude in each tap, respectively. For $L = 1$, one has $\gamma^{TR} = 0$; with increasing L it converges to 1. Hence, in the time-reversal channel, the ratio of the energy of the zeroth tap to the sum of the energy of all the other taps converges to a fixed value.

If $h[l]$ is a complex Gaussian random variable with mean 0 and variance $\langle E_0 + E_S \rangle = \langle E_H \rangle$, we have $\beta_2 = \langle E_H \rangle$ and $\beta_4 = 2\langle E_H \rangle^2$. Hence,

$$\gamma_{TR} = \frac{2L\langle E_H \rangle^2}{2\langle E_H \rangle^2 + (L-1)\langle E_H \rangle^2} - 1 = \frac{2}{1 + \frac{1}{L}} - 1. \tag{7}$$

For large L, the power in the tails of the impulse response becomes about equal the power in the peak. Hence, even though ISI can be significantly suppressed, any system without equalization operates at best at 0 dB signal to ISI plus noise ratio. For small L, the compression works in the mean better than for a large one, even though the difference quickly levels out. But the probability that a channel realization occurs that does not focus at all (i.e., all channel taps are real and have the same magnitude) is much larger than for long sequences.

C. Signal Model and TR-MMSE Equalizer

The transmitter modulates symbols s_k using a pulse shaping filter $\phi(\tau)$ of bandwidth $B = 1/T_s$. It then performs TR on the pulse stream. Thus:

$$x(t) = \sum_k s_k \phi(t - kMT_s) \otimes \tilde{h}^*(-t) \tag{8}$$

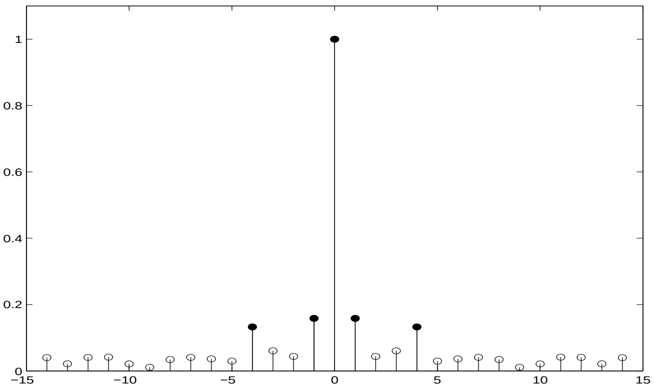

Fig. 1. Schematic of TR impulse h_{TR} and truncated TR CIR h_{TR}^s. In this toy example $L = 29$, $L_s = 9$ and the number of effective taps (marked as filled circles) is 5.

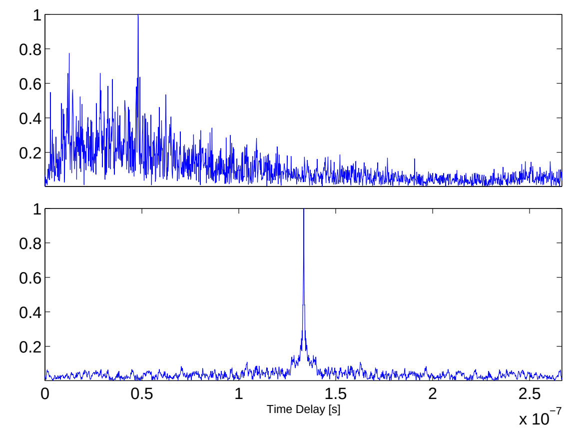

Fig. 2. Impulse Response of the channel (above) and of the time-reversed channel (below)

where $\tilde{h}(\tau)$ is the (infinite bandwidth) CIR. After matched filtering the received signal can be written as:

$$y(t) = \sum_k s_k \phi(t - kMT_s) \otimes \tilde{h}^*(-t) \otimes \tilde{h}(t) \otimes \phi^*(-t) + n(t) \tag{9}$$

The receiver then samples $y(t)$ at lMT_s:

$$y[l] = y(lMT_s) = \sum_k s_k h_{TR}[l - k] + n[l], \tag{10}$$

where $h_{TR}(t) = h(t) \otimes h^*(-t)$, $h(t) = \tilde{h}(t) \otimes \phi(t)$, and $h_{TR}[k] = h_{TR}(kMT_s)$. The sampled received signal is thus the convolution of the symbols with a downsampled-by-M version of $h_{TR}(.)$.

Due to the temporal focusing described above the equalizer at the receiver does not have to use the quite long (time-reversed) CIR. Instead the equalizer uses a "shortened" TR CIR by keeping only a few taps which capture most of the energy of h_{TR} and still achieve very good performance. More precisely, we construct a shortened TR CIR h_{TR}^s of length L_s as illustrated in Fig. 1 by defining

$$h_{TR}^s[l] := \begin{cases} h_{TR}[l] & \text{if } |h_{TR}[l]| \geq \delta, \\ 0 & \text{else,} \end{cases} \tag{11}$$

for $|l| \leq L_s/2$ and $L_s = \max\{l : |h_{TR}[l]| \geq \delta\}$. Here δ is an a priori chosen tolerance level that depends on the target SNR. For convenience we will rename the indexing of h_{TR}^s such that $h_{TR}^s = [h_{TR}^s[0], \ldots, h_{TR}^s[L_s - 1]]^*$.

The symbols are transmitted one block at a time with a guard period of length L_s. The output of the channel is:

$$y[m] = (x \otimes h_{TR})[m] + n_R[m]$$

Here $x[m]$ is the input signal $m = 0, \ldots, N - 1$ and $y[m]$ is the output signal $m = 0, \ldots, N + L - 2$. In matrix notation this can be written as Toeplitz-type system

$$\mathbf{y} = \mathbf{H}_{TR}\mathbf{x} + \mathbf{n}_R$$

where $\mathbf{y} = [y[0] \ldots y[N + L - 2]]^T$ and $\mathbf{x} = [x[0] \ldots x[N - 1]]^T$. Using h_{TR}^s instead of h_{TR} for the MMSE equalizer we replace $\mathbf{H}_{TR}$ by the matrix $\mathbf{H}_{TR}^s$ of size $(N + L_s - 1) \times N$ given by

$$\mathbf{H}_{TR}^s = \begin{bmatrix} h_{TR}^s[0] & 0 & \ldots & 0 \\ \vdots & & & \\ h_{TR}^s[L_s - 1] & & \ddots & \vdots \\ 0 & & & 0 \\ & \ddots & & h_{TR}^s[0] \\ \vdots & & & \vdots \\ 0 & \ldots & 0 & h_{TR}^s[L_s - 1] \end{bmatrix},$$

and the matrix equation for the MMSE estimator becomes

$$\hat{\mathbf{x}} = \left((\mathbf{H}_{TR}^s)^* \mathbf{H}_{TR}^s + \frac{1}{SNR_{RX}} \mathbf{I} \right)^{-1} (\mathbf{H}_{TR}^s)^* \mathbf{y}.$$

Assuming the transmitted symbols are from a BPSK alphabet, we can now decode $\hat{\mathbf{x}}$ by looking at the sign of each element.

We note that $\mathbf{H}_{TR}^s$ is a sparse Toeplitz matrix, thus $\hat{\mathbf{x}}$ can be computed efficiently by combining sparse matrix techniques with fast Toeplitz solvers [11].

III. Simulations

A. Measurement Data and Temporal Focusing

Measurements were conducted with a network analyzer by Intel Corp. at off-peak hours to ensure channel stationarity. The environment is an office space (40m $\times$ 60m) with many cubicles; measurements were conducted at several locations. They span a bandwidth of 2-8 GHz with 3.75 MHz frequency resolution. Antennas are vertically polarized. The measurements are corrected to compensate for the system components (including cable, gain stages, and antennas). The height of the transmit antenna is about 2.5m and that of the receive antenna is 1m above the floor. As a sample result, Fig. 2 shows the CIR (above) and the time-reversed CIR (below). The compression is clearly visible. Still, this figure is not suitable to tell the ratio between the power in the peak of the time-reversed impulse response and that in the tails. In

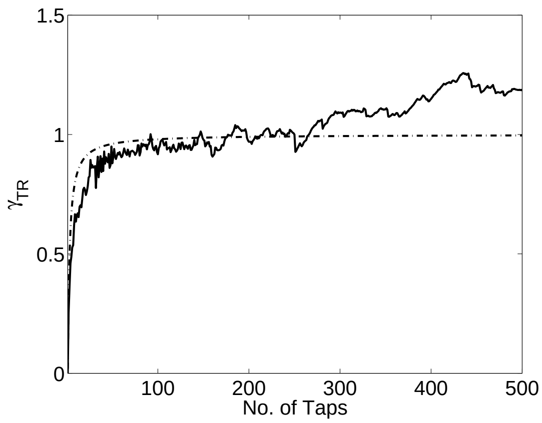

Fig. 3. In the TR channel, the ratio between the eneregy in the main tap and the energy in the tail of the CIR converges for independent taps to a fixed ratio.

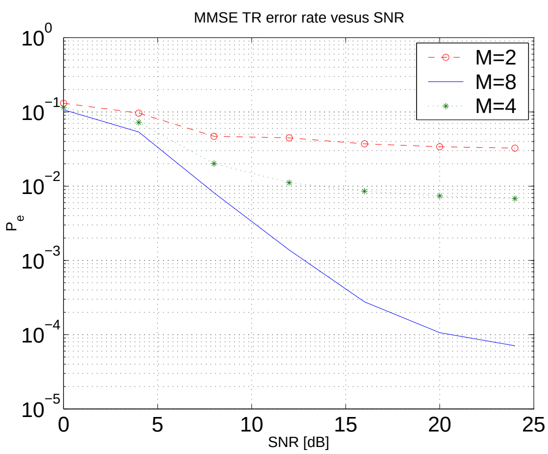

Fig. 4. BER vs SNR when the receiver only estimates the 20 largest taps of the channel. Bandwidth expansion factor of 8 (solid line), 4 (dotted line), and 2 (dashed line)

order to demonstrate this ratio, we plotted in Fig. 3 the time compression ratio γ_{TR} for a CIR length between 1 and 500 taps, where the entire length of the CIR is fixed at 1/3.75MHz. The curve was actually computed in frequency domain; in order to suppress fluctuations due to the randomness of the samples, a sliding window was implemented, so that γ_{TR} was actually averaged over all available frequency samples.

For a small number of taps, the curve coincides very well with the theoretical one. At about 200 taps, the maximum theoretical value of 1 is exceeded; beyond that, correlation between the taps starts to play a role and the power in the tails of the CIR will be stronger than that in the main peak.

B. Performance of the Equalizer

We used the single channel realization whose magnitude response at full bandwidth, B, is shown in Fig. 2. Three cases were considered. Case I has $M = 8$ (see Eqn. 10) and bandwidth B. Case II has $M = 4$ and bandwidth $B/2$. Finally

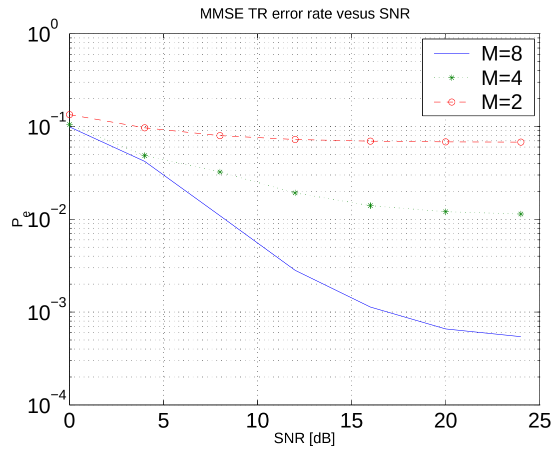

Fig. 5. BER vs SNR when the receiver only estimates the 10 largest taps of the channel. Bandwidth expansion factor of 8 (solid line), 4 (dotted line), and 2 (dashed line)

case III has $M = 2$ and bandwidth $B/4$. The constellation was BPSK and no channel coding was used. Thus all the scenarios have the same data rate and all channels have 250 taps. Fig. 4 shows the bit error rate (BER) curves for these scenarios when the receiver only estimates the 20 strongest taps. When $M = 2$ the curve floors very rapidly since not a very significant portion of the energy of the CIR is captured by the largest 20 taps. When $M = 4$, the ISI energy is due only to every fourth tap of the TR channel. Thus more of the energy of the effective CIR is captured by the strongest 20 taps. This effect improves significantly in the case where $M = 8$. Note that the case $M = 8$ uses more bandwidth. However, the BER floors at a reasonable SNR of 15 dB for UWB. Fig. 5 shows the BER curves for the same three cases but when the receiver estimates only 10 strongest taps of the CIR. In this scenario the error flooring occurs earlier for all the cases. However, the complexity of the receiver is significantly reduced.

Fig. 6 plots the BER for a different CIR. Here the receiver estimates the largest 20 taps of the CIR. Comparison between this figure and Fig. 4 shows that the BER performance is very similar for these two channel realizations. This is due to the large diversity available in this ultra-wideband channel.

Fig. 7 shows the BER vs number of strongest CIR taps that the receiver estimates for equalization at 15 dB SNR. This plot shows that the BER reduces approximately exponentially with increasing number of taps. Fig. 8 shows the BER vs number of strongest CIR taps that the receiver estimates at 15 dB SNR when the transmitter does not use TR. Figs. 7 and 8 demonstrate the temporal focusing ability of TR. The receiver requires to estimate much less taps with TR than without it in order to achieve the same BER performance.

Note that the plots are for the strongest taps. These taps may not be next to one another. There may be zeros between any two consecutive strongest taps. Thus the total length in samples of the estimated channel is in general larger than the number of strongest taps estimated. However, the complexity

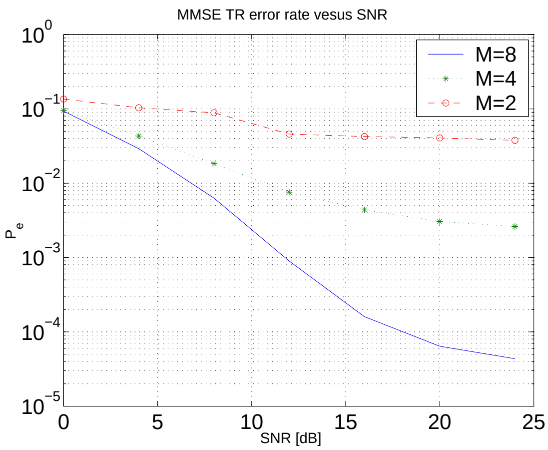

Fig. 6. BER vs SNR when the receiver only estimates 20 largest taps of the channel - another channel realization. Bandwidth expansion factor of 8 (solid line), 4 (dotted line) and 2 (dashed line).

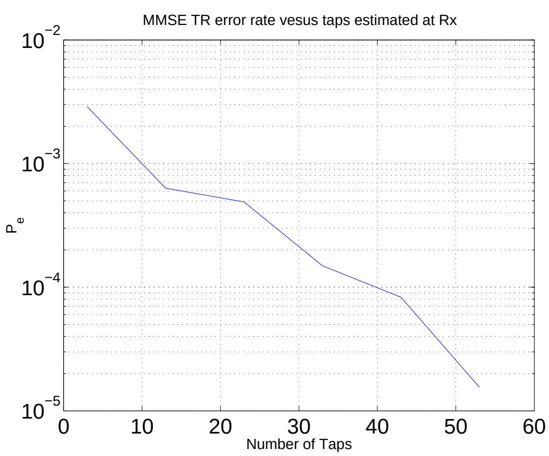

Fig. 7. BER vs number of strongest taps estimated at the receiver. SNR = 15 dB

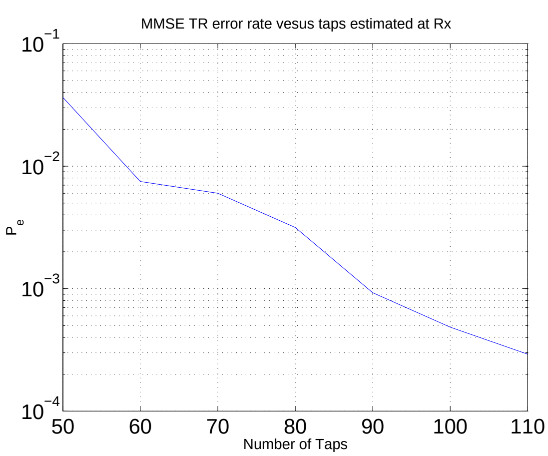

Fig. 8. BER vs number of strongest taps estimated at the receiver with regular transmission. SNR = 15 dB

of equalization is approximately the same as if the length of the estimated channel is equal to the strongest number of taps estimated.

IV. Conclusion

Time reversal significantly reduces the number of taps that the receiver needs to estimate in order to achieve a certain target BER. If the CIR has iid complex Gaussian taps the ratio of signal to ISI power approaches unity as the length of the CIR increases without bound. We show that this ratio is very close to unity when the CIR is more than about 10 taps long.

In order to mitigate the ISI one can fix the bandwidth and increase symbol spacing by an integer factor of M. The larger M, the less taps of the CIR the receiver needs to estimate in order to attain a reasonable BER. However, larger M means more wasted bandwidth. With no channel coding and by only estimating the 20 strongest taps out of approximately 250 channel taps, the receiver can reach 10^{-3} BER at around 12 dB SNR with MMSE equalization and $M = 8$. In order to achieve such a performance without TR, the receiver needs to estimate almost all the taps of the CIR.

References

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works," *IEEE Commun. Lett.*, vol. 2, no. 2, pp. 36–38, Feb. 1998.

[2] ——, "On the energy capture of ultrawide bandwidth signals in dense multipath environments," *IEEE Commun. Lett.*, vol. 2, no. 9, pp. 245–247, Sept. 1998.

[3] ——, "Characterization of ultra-wide bandwidth wireless indoor channels: A communication-theoretic view," *IEEE J. Select. Areas Commun.*, vol. 20, no. 9, pp. 1613–1627, Dec. 2002.

[4] D. Rouseff *et al.*, "Underwater acoustic communication by passive-phase conjugation: Theory and experimental results," *IEEE Journal of Oceanic Engineering*, vol. 26, pp. 821–831, 2001.

[5] M. G. Heinemann, A. Larazza, and K. B. Smith, "Acoustic communications in an enclosure using single-channel time-reversal acoustics," *Appl. Phys. Lett.*, vol. 80, pp. 694–696, 2002.

[6] G. F. Edelmann *et al.*, "An initial demonstration of underwater acoustic communications using time reversal," *IEEE Journal of Oceanic Engineering*, vol. 27, pp. 602–609, 2002.

[7] A. Derode *et al.*, "Taking advantage of multiple scattering to communicate with time-reversal antennas," *Phys. Rev. Letters*, vol. 90, 2003.

[8] A. Derode, P. Roux, and M. Fink, "Robust acoustic time reversal with high-order multiple scattering," *Phys. Rev. Letters*, vol. 75, pp. 4206–4209, 1995.

[9] S. M. Emami, J. Hansen, A. D. Kim, G. Papanicolaou, A. J. Paulraj, D. Cheung, and C. Prettie, "Predicted time reversal performance in wireless communications using channel measurements," *IEEE Communications Letters*, 2004, accepted for publication.

[10] J. D. Choi and W. E. Stark, "Performance of ultra-wideband communications with suboptimal receivers in multipath channels," *IEEE J. Select. Areas Commun.*, vol. 20, no. 9, pp. 1754–1766, Dec. 2002.

[11] G. Golub and C. van Loan, *Matrix Computations*, 3rd ed. Baltimore: Johns Hopkins, 1996.