Communications in High Delay Spread-Bandwidth Channels

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Motivation

- Future comm systems must support large data rates and high user densities
- Large rates and low interference levels can be achieved using multiple antennas (MA) at both transmitter and receiver (MIMO communications)
- Or, equivalently, by increasing the bandwidth
- Considerable work on MIMO comm systems has been done, but the application of MA is too costly
- Comm systems in high delay spread-bandwidth (HDSBW) channels (*e.g.* UWB) have received less attention
- HDSBW channels also have good interference reduction properties which are not widely appreciated

Challenges & Benefits of HDSBW Channels

- When the environment is richly scattering, increasing the bandwidth results in a channel impulse response with many channel taps
- There are benefits and challenges associated with such channels:
  - **Benefits:**
    - Spatial focusing resulting in interference suppression
    - Time compression resulting in reduced intersymbol interference (ISI)
    - Diversity gain resulting in reduced fading
  - **Challenges:**
    - Non-trivial channel estimation and modeling
    - Complex equalization at Tx and/or Rx to implement time compression
    - Complex beamforming at Tx to implement spatial focusing

Background — TR Communications

- The group's focus was initially on TR-transmission
- TR is a transmission scheme designed for high-delay spread-bandwidth (HDSBW) channels
- The transmit signal is precoded such that only at one point in space and time the scattered components of the signal superimpose coherently
- At any other point, the received signal looks like noise (incoherent superposition)
- Origin of TR is in underwater acoustics and ultrasound

Historical Background

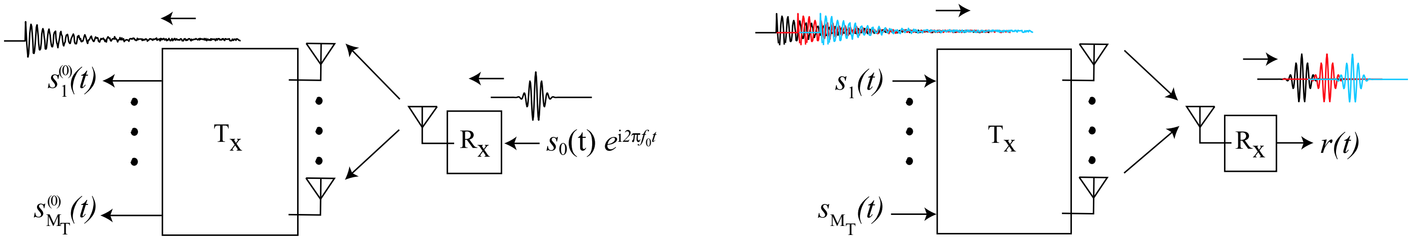
- **Ultrasound and underwater sound:**
  - Spatial focusing seen in TR experiments (without communications) in the last 15 years, in ultrasound (Fink, Paris) and in underwater sound (Kuperman, UCSD).
  - Theory for spatial focusing in TR in random media carried out by Jackson and Dowling (1990+), Fink (1995+), Kuperman, and the Stanford Math Group (2002).
  - TR communication schemes demonstrated by Kuperman (underwater sound, 2002), Rouseff (passive underwater sound, 2001), Fink (ultrasound, 2003, and EM, 2004), and Larazza (underwater sound, 2002). Space focusing and time compression of signals seen.
- **Wireless**
  - We demonstrated spatial focusing and time compression gain at Stanford in Dec 2003 using UWB measurements made by Intel.
  - Aalborg university has shown spatial and temporal focusing gain using TR with wideband outdoor measurements

Overview

- A brief introduction to TR
- Computational channel modeling
  - Computational resources
  - TR simulations for various propagation models
- Channel measurements
  - Computed TR
  - TR sounder planned measurements
- Communication system design issues
  - Single user (SU) communications
  - Muli-user (MU) communications
- Conclusions and outlook

A Brief Introduction into TR

- Consider a MISO system at very high SNR
- Rx sends a pilot to Tx before transmission



- Tx sends the message convolved with the time reversed channel impulse response (CIR).
- The received signal before noise is

$$r(t) = \sum_k h_k(t) * h_k^*(-t) * s(t)$$

$s(t)$: transmitted signal, $h_k(t)$: channel impulse response, $r(t)$: received signal

- This achieves focusing in space and time at Rx

The Merits

We benefit from

1. Diversity enhancement for single antenna receivers allows for reduced fading
2. Time compression allows for simple receivers in the downlink
3. Spatial focusing allows for interference suppression

Channel Modelling for HDSBW Channels

- Channel models serve to characterize statistically and simulate realistically real-world propagation conditions
- There exist widely used channel models for current systems
  - Narrowband
  - Wideband
    - 3G systems: 5MHz
    - WLAN : 20MHz
  - UWB channel models are being developed, but are not uniformly accepted.

Current Channel Model Parameters

- Parameterization of the environment
  - indoor/outdoor,
  - urban/suburban/rural
- Power dependence on distance between Tx and Rx
- Delay spread dependence on distance
- Power delay profile
- Local signal distribution
  - Rayleigh/Ricean (K-factor)
- Directional information necessary for MISO/MIMO operation

Research Opportunities in HDSBW Channels

Environments	• How is propagation different in residential/ office environments? • How do we differentiate LOS and NLOS situations? • Do we need UWB characterization of outdoor environments?
Power	• What is the power roll-off law in LOS/ NLOS situations? • What is the range of large delay spread systems? • How do we account for deterministic propagation effects such as waveguiding?
Delay Spread	If at short distances only a limited number of paths is received, how does the delay spread depend on the distance from the transmitter?
Power delay profile	What are realistic power delay profiles (exponential/ clustered)?
Signal Distribution	• As paths are more clearly resolved, are the channel taps then Rayleigh or Ricean distributed? • What is the K-factor?
Angular information	How does directional channel information depend on the distance from the transmitter?
Tap Correlation	Given the high path resolution, how correlated are paths that arrive close to each other?

Computational Resources: Hardware

Computer Cluster “doozers”:

- Master node
  - 4×3.06 GHz Intel Xeon, 512 KB Cache, 4 GB of RAM with ~ 260 GB of hard drive space
- 16 compute nodes
  - 2×3.06 GHz Intel Xeon, 512 KB Cache, 4 GB of RAM
- Switch
  - 1 GB Ethernet

Computational Resources: Performance

Random waveguide simulations

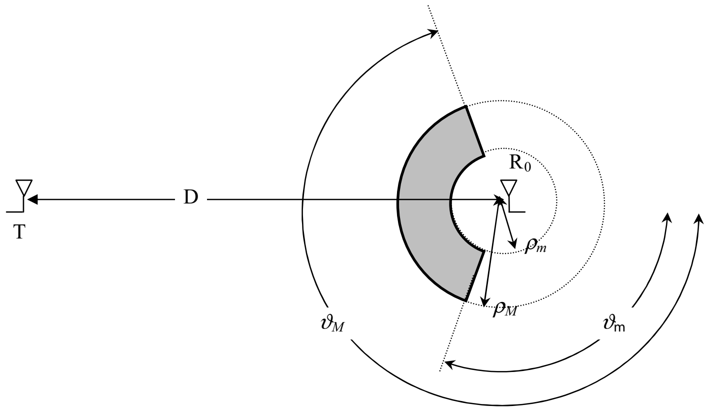
- A 2D phase screen code is fully operational. It incorporates several features important in channel modeling
  - Propagation distances on the order of 10^5 wavelengths
  - Multiple scattering due to inhomogeneities in the medium and boundaries
  - Dissipation
  - Angle diversity control
  - Frequency diversity control
- Realistic 2D calculations take only 40 seconds on the computer cluster (core computation is only 10 seconds). They take 480 seconds on a single workstation.
- A 3D long distance wave propagation code is near completion (testing stage).

Present Work in Channel Modeling

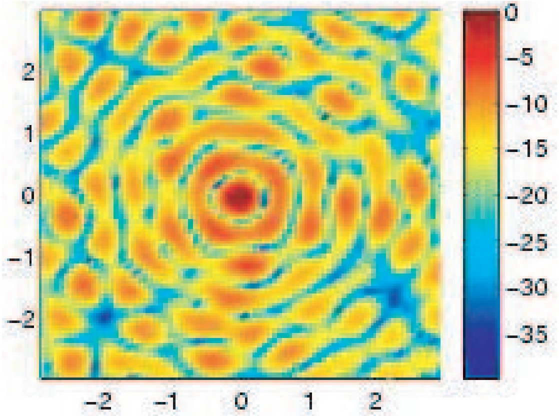
PHYSICS-BASED MODELS

- Develop realisitic models of wave propagation in richly scattering environments that bridge the gap between parametric channel models and physical experiments.
- Simulations using these models allows for the modeling of space-time focusing in time reversal.
- Some examples include
  - Discrete scatterer model for outdoors (C. Oestges)
  - Waveguide model for urban canyons and indoor corridors (A. Kim)
  - Enclosures for indoors (D. Berebichez)

Channel Modeling: Discrete Scatterer Model



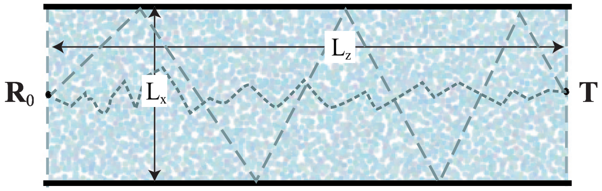
Large delay spread is due to discrete "point" scatterers are distributed randomly in an annulus that surrounds the receiver.



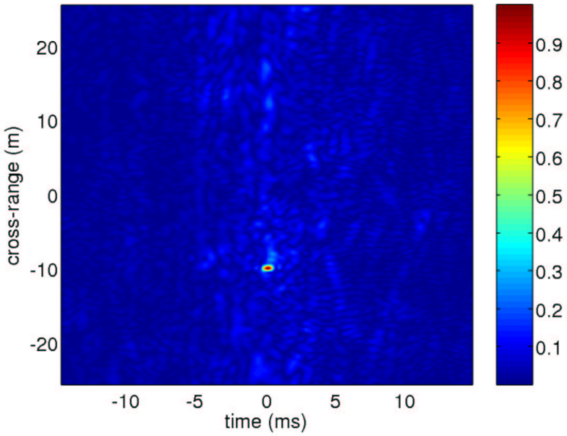
The spatial variation of the intensity at the time of refocus (100 MHz bandwidth centered at 2.5 GHz)

Using this model we have characterized space-time focusing of broadband signals and its dependence on bandwidth, delay spread, angle spread and number of antennas (to appear in IEEE Trans. Ant. Prop.).

Channel Modeling: Waveguide Model



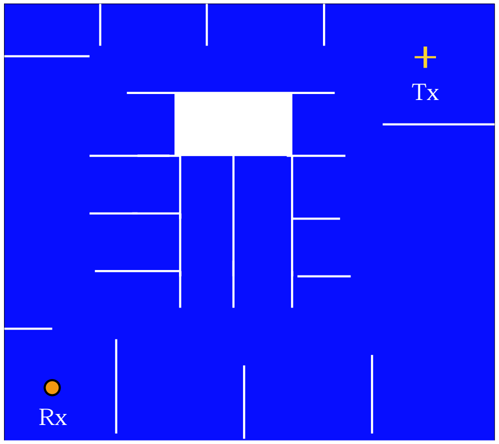
Large delay spread in the waveguide is due to reflections from the walls as well as random inhomogeneities inside.



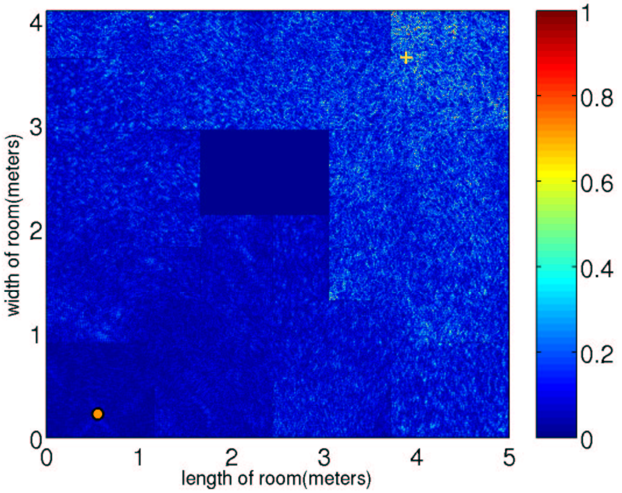
The refocused intensity as a function of cross-range (y-axis) and time (x-axis) at the receiver plane.

We are using this model to perform direct numerical simulations that evaluate time reversal effects as they relate to communication system performance including intersymbol interference, co-channel interference and transmission rate (to appear in IEEE J. Ocean. Eng.).

Channel Modeling: Enclosure Model



Large delay spread is due to boundaries (e.g. office walls) within the enclosure.



The refocused intensity at the refocus time.

We are using this model to perform direct numerical simulations to study time reversal effects in broadband indoor wireless systems (D. Berebichez's thesis work).

Channel Measurements

- Computed TR
  - Outdoor low DSBW MIMO measurements - Toronto data
  - Outdoor high DSBW MIMO measurements - Helsinki data
  - Indoor high DSBW UWB measurements - Intel data
- The Stanford TR sounder

Outdoor Low DSBW MIMO Measurements - Toronto Data

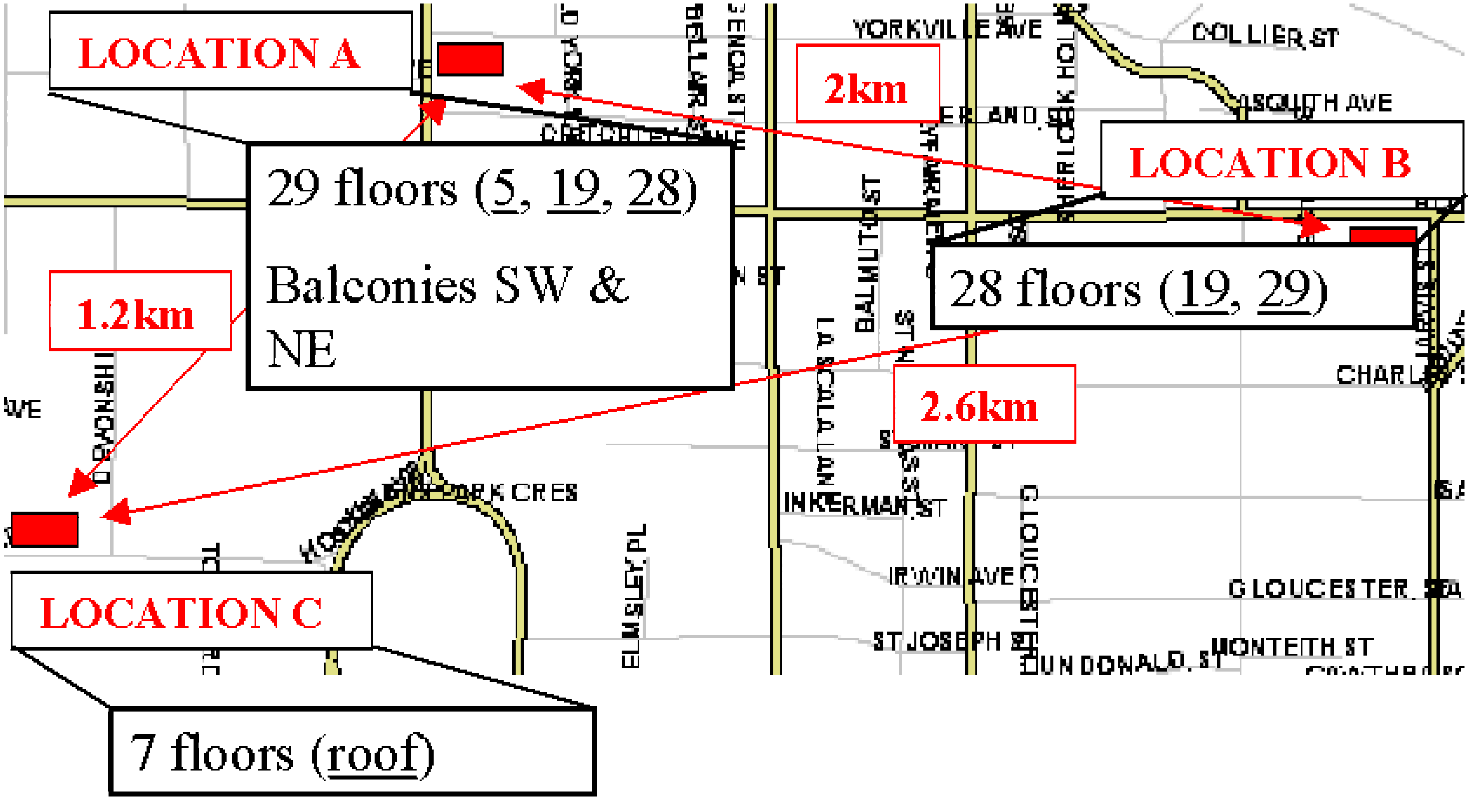
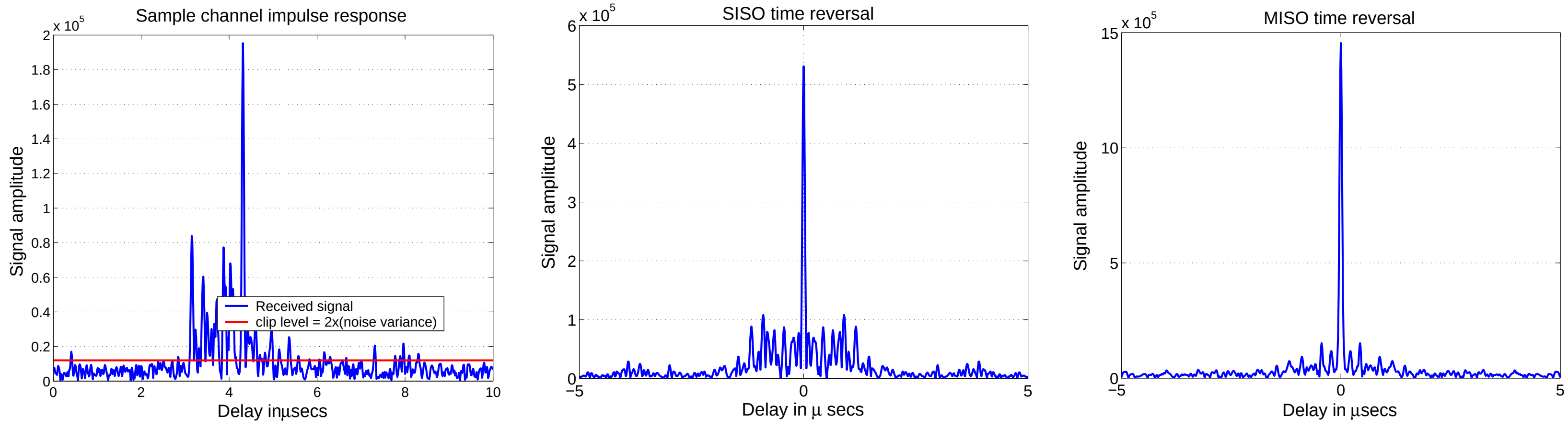


Figure 1:

- $f_c = 5$ GHz, BW= 20MHz (DSBW≈ 20)
- 8 element uniform linear arrays, $s = \lambda/2$

Visual Inspection of Computed TR

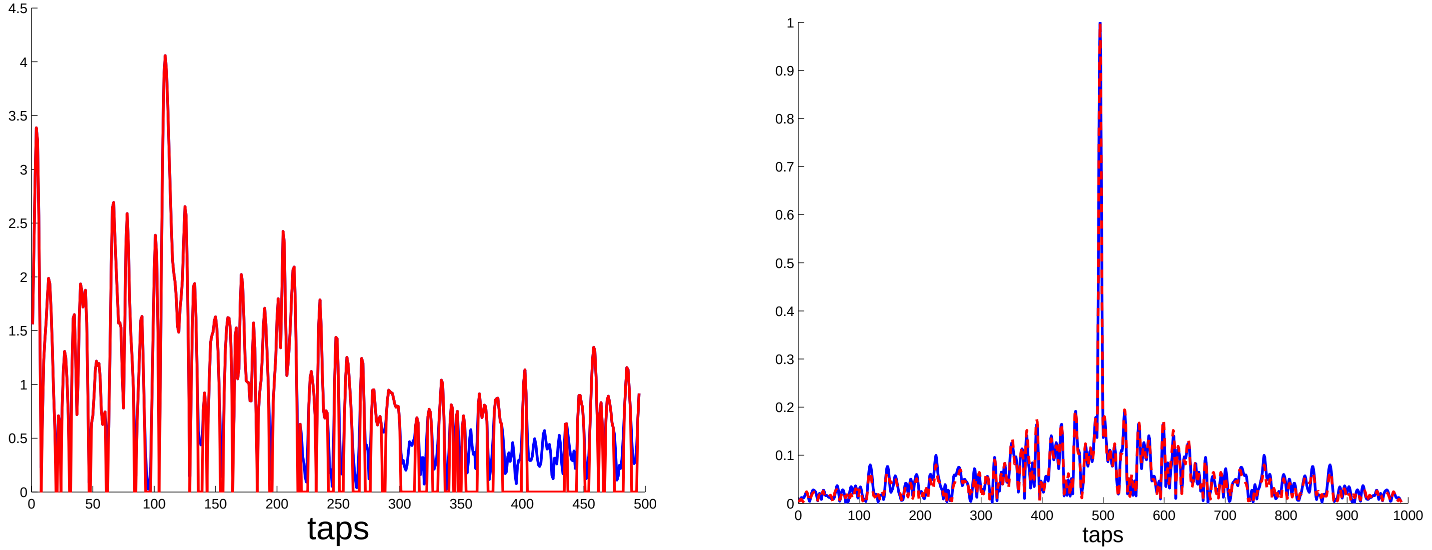


- left: SISO communications in delay spread channel
- middle and right: TR and MISO TR with 8 Tx antennas

(Kyritsi et al, IEEE Antennas and Wireless Propagation Letters, 2004).

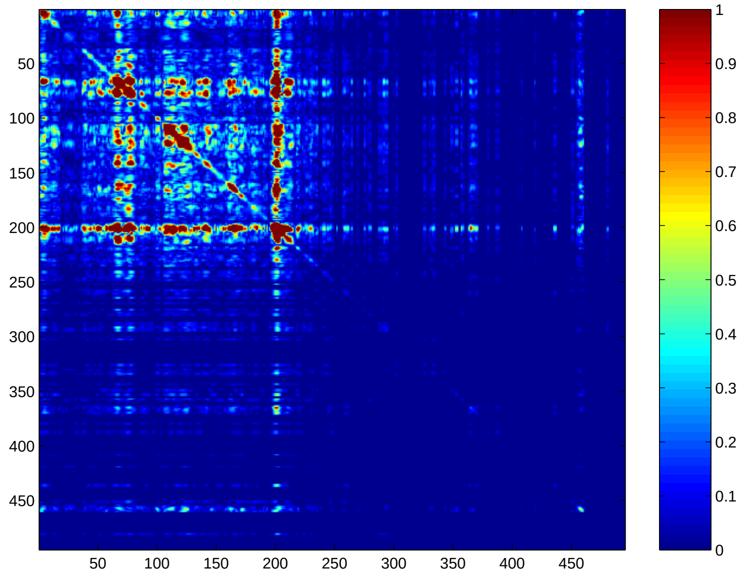
Nokia Measurements

- Computed TR with 100 MHz data, downtown Helsinki
- DSBW$\approx$ 100 — data focuses very nicely



(manuscript in preparation)

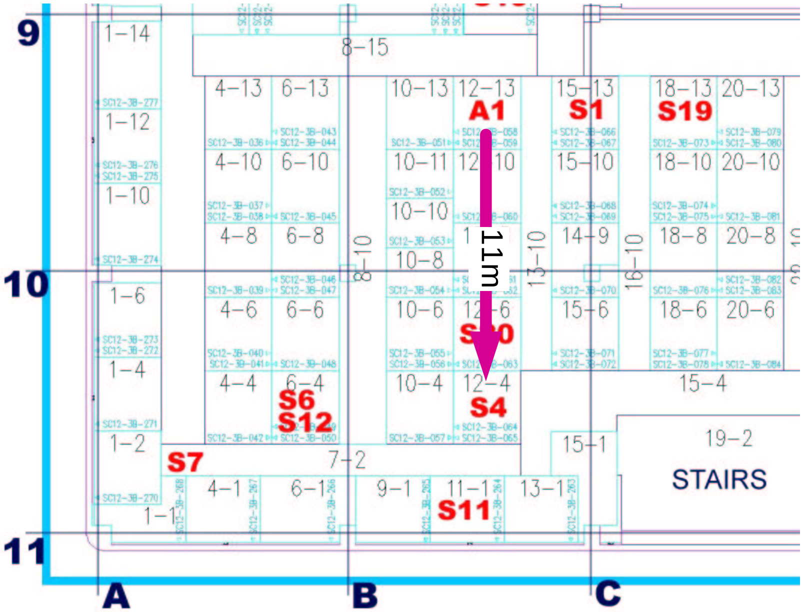
Nokia Measurements — The Imperfections in Real Data



- TR works best when tap coefficients of different delay are independent
- For these outdoor measurements however this is not the case: $r_{ij} = \langle h(\tau_i)h^*(\tau_j)\rangle$
- r_{ij} for the data is shown in the plot

(manuscript in preparation)

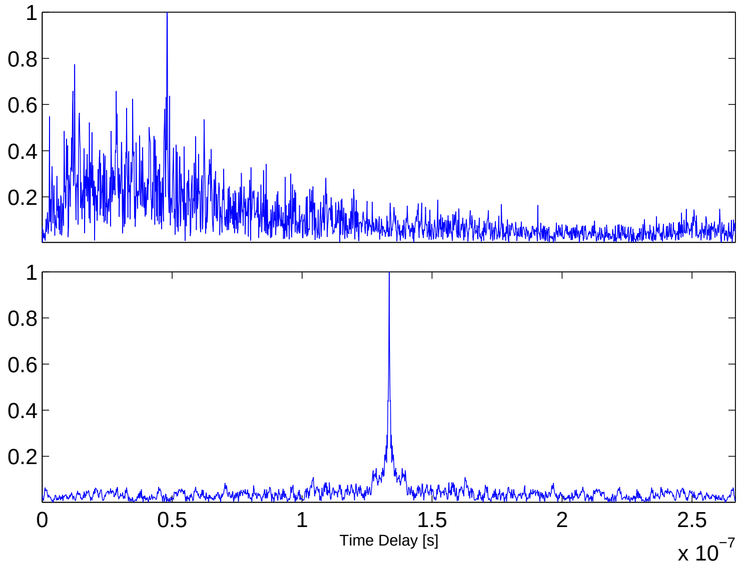
Intel UWB Measurements



- Transmission band 2-8 GHz ($\lambda_0 = 6\ cm$)
- Coherence bandwidth 20 MHz ($DSBW = 300$)
- Line-of-sight measurements obstructed by cubicle partitions

(to appear in IEEE Communication Letters)

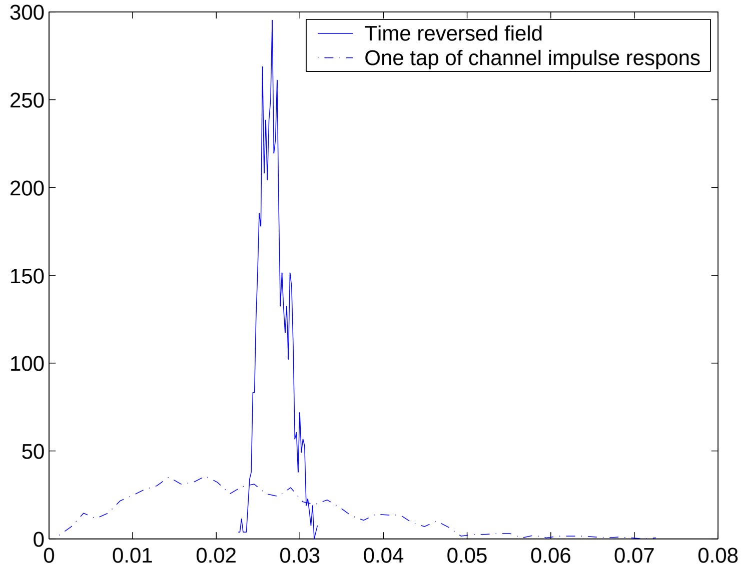
Intel UWB Measurements (2)



- In this very long channel we observe very good compression
- But we also receive a very long ringing signal

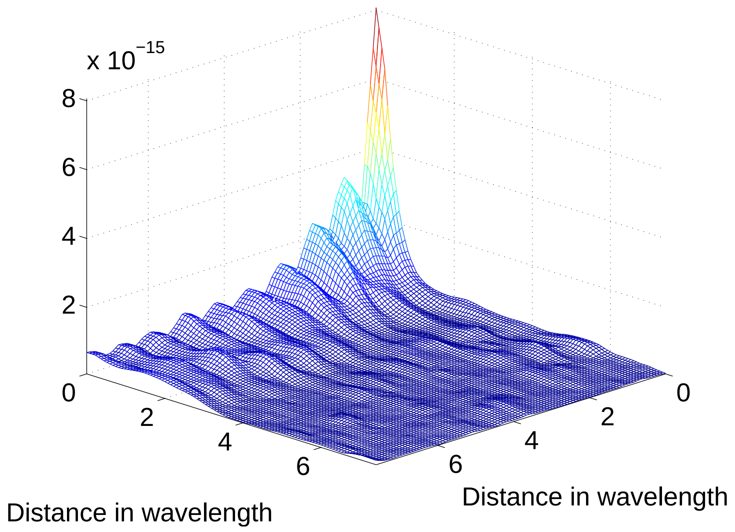
(to appear in IEEE Communication Letters)

Channel Hardening



- Pre-equalization on long DS channels captures diversity
- The pdf of received power at the central tap is much narrower if the transmit signal is preequalized

Spatial Focusing



- The one-shot transmitted signal is well focused around the intended receiver

(to appear in IEEE Communication Letters)

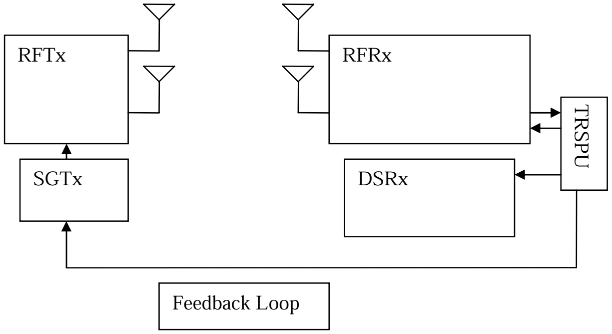
TR Sounder Experiments

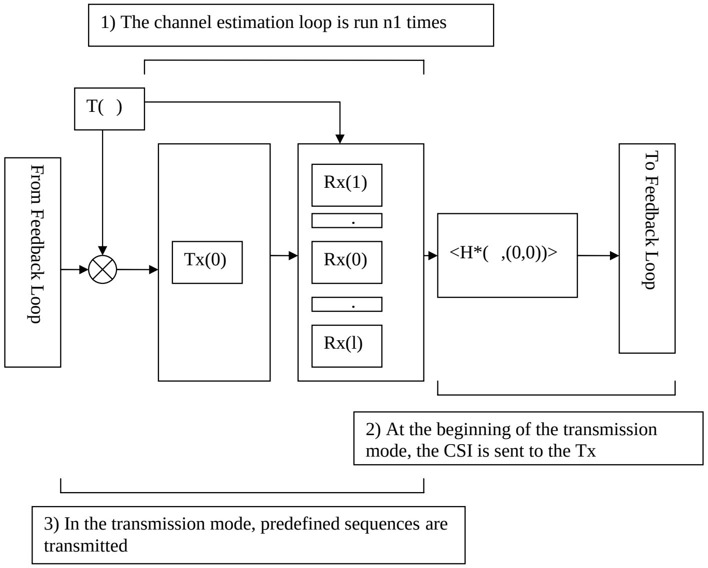
- The Stanford TR group is designing a TR channel sounder in cooperation with the German company MEDAV
- The goal is to investigate and demonstrate TR in real-world environments
- We shall examine the impact of delay spread & bandwidth, antenna type & angular spread, temporal variation of the channel and feedback of channel state information (CSI), etc.

Technical Specifications

- Carrier frequency 2.450 GHz, 120/240 MHz bandwidth
- 33 dBm transmit power
- 1×8 SIMO system, where TR is performed to one intended receiver
- The other receivers can be used to demonstrate spatial focusing
- Channel state information feedback via Ethernet/WLAN or optical fiber

Block Diagrams





1) The channel estimation loop is run n1 times

2) At the beginning of the transmission mode, the CSI is sent to the Tx

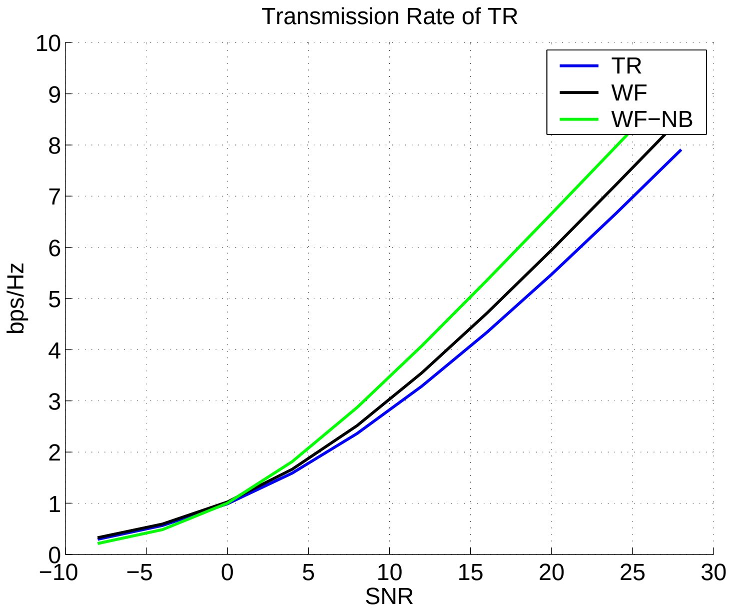
3) In the transmission mode, predefined sequences are transmitted

- The sounder's core is a MIMO channel sounder with added time reversal signal processing unit (TRSPU).
- The sounder has channel estimation mode & time-reversal mode

Communication System Design Issues in HDSBW Channels

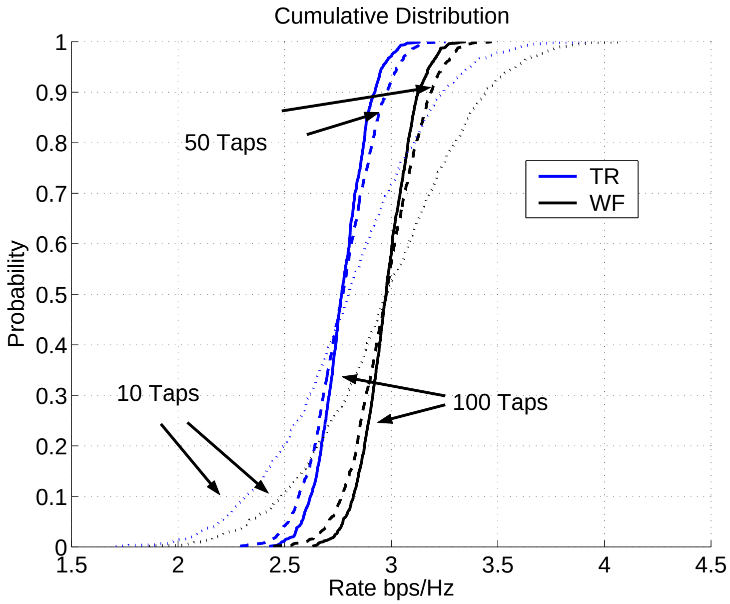
- Information rate of HDSBW TR systems
- Equalization: transmit preequalization and receive equalization
- Channel estimation and partial CSI
- Spatial focusing and interference suppression
- MU communications with strong interference suppression

Single User Transmission Rate



- Achievable transmission rate with various power allocation techniques (WF: water-filling, WF-NB: water-filling on an ISI free channel)
- Results for capacity of delay spread channels apply (Hirt & Massey, 1988)
- TR implies the power allocation scheme $P(\omega) = |H(\omega)|^2$
- At low SNR, TR becomes optimal

Channel Hardening



- Outage distributions are shown for simulated channels of several tap lengths.
- TR and water filling (WF) algorithms are compared.
- As the DSBW of the channel increases, the capacity tends toward a deterministic value (*i.e.* the cdf becomes steeper).
- Transmission over delay spread channels improves outage capacity.

Research Opportunities

- Information rate not very suitable to characterize TR systems
- Information rate implies infinitely complex transmitters and receivers
- TR is a suboptimal scheme that reduces complexity
- Future work:
  - Define information rates for systems of reduced complexity
  - Single user and multi user channel capacity with partial CSI

Equalizing Channels with High Delay Spread

- Most well-known equalizers have complexity that grows as a function of # of channel taps (e.g. Qureshi 1984), *e.g.*,
  - # of taps in linear equalizers (LEs) and decision feedback equalizers (DFEs) is a linear function of # of channel taps
  - # of states in a maximum likelihood sequence estimator (MLSE) is an exponential function of # of channel taps

Equalizing Channels — Known Results

- Problem:
  - Time-domain LEs and DFEs with large # of taps require heavy computations and adapt slowly
  - MLSEs with large # of states are too complex to build
- Various reduced-complexity equalization techniques also exist, *e.g.*,
  - Apply channel shortening filters before equalization (see Al-Dhahir & Cioffi 1996 for a brief summary)
  - Reduced-state sequence estimators (Eyuboglu & Qureshi 1988; Duel-Hallen & Heegard 1989)
  - Reduced-search algorithms for MLSEs (M-algorithm (see Anderson & Mohan 1984 for overview), T-algorithm (Simmons 1990), etc.)
- Most techniques are *not* intended for HDSBW channels with hundreds of taps, especially when energy of taps at the tail is significant.

Research Opportunities in Equalizing HDSBW Channels

- Equalization strategies that work well for HDSBW channels are currently very limited. Two popular approaches:
  - Frequency domain implementation of LEs and DFEs (see Falconer *et. al.* 2002 for overview)
  - Perform equalization at transmitter rather than receiver where processing power is less stringent (e.g., Lee & Zhuang 2003; Choi & Murch 2004)
- Equalization for systems where HDSBW channels are dominant has not been studied. Current standards for ultra-wideband (UWB) systems are based on transmission formats that avoid equalization as in pulse-positioned modulation (PPM) or orthogonal frequency division multiplexing (OFDM).

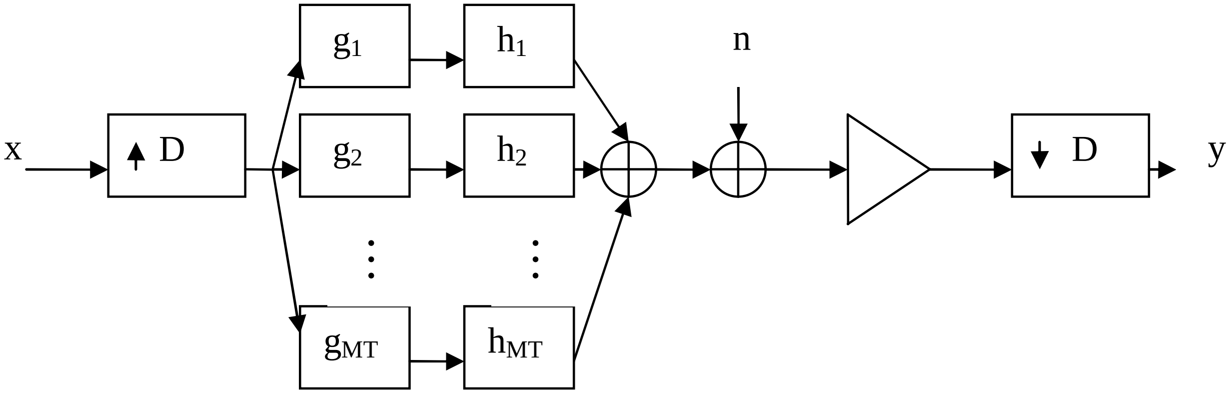
The Group's Current Work on Equalization

- Optimal transmit/receive linear equalization schemes — simple receivers
- Performance of 1-tap receivers with rate back-off
- Rate back-off and receive equalization - Intel UWB data
- Channel shortening with TR - Toronto data
- Rate improvement with TR - Toronto data

Optimal Tx/Rx Processing

- TR shortens the channel to suppress ISI
- Derive the optimal transmission scheme for a transmitter with high complexity, and a receiver with only a single tap
- This scheme implies that the transmitter has the maximum burden wrt temporal focusing
- Receiver does not even need to estimate the channel

Block Diagram for MISO System with Optimal Prefilter



- The optimal prefilter is

$$\hat{\mathbf{g}}_{\beta,N} = \frac{1}{\beta}\left(\mathbf{H}^H\mathbf{H} + \frac{\sigma^2}{P_{Tx}}\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{e}_\Delta$$

  $\mathbf{H}$: Toeplitz channel matrix, P_{Tx}: transmit power, σ^2: noise

- β normalizes transmit prefilter
- TR emerges as optimal at low SNR, but causes ISI compared to the optimal prefilter

Rate back-off to combat ISI

- Signal model of the rate back-off scheme is

$$y^{[D]}[k] = \beta \sum_{i=1}^{M_T} \sum_{l} (h_i * g_i)[Dl] x[k-l] + \beta n[k].$$

x: transmit symbol, y: receive symbol, n receive noise

h_i: channel impulse response, g_i prefilter, D: rate-back off, M_T: no. of transmit antennas, $*$: convolution

- We introduced rate-back off by a factor D to combat ISI: data transmission rate is not inverse bandwidth, but a factor of D lower than that
- This classifies TR as a system with excess bandwidth

Performance Evaluation

- Define the effective SNR ρ_{eff} of the scheme as

$$\rho_{\text{eff},D} = \frac{P_{Tx}E[P_0]}{P_{Tx}E[Q^{[D]}] + \sigma^2} = \frac{E[P_0]}{E[Q^{[D]}] + \frac{1}{\rho}}$$

  $E[P_0]$: mean power in the peak, $E[Q^{[D]}]$: mean power of the ISI for rate-back off D, P_{Tx}: transmit power, σ^2: noise

- $\gamma_{TR} = E[P_0]/E[Q^{[D]}]$ is the focusing of the signal at the receive antenna
- Compare against ρ_{MFB},

$$\rho_{\text{MFB}} = \frac{P_{Tx}E[\sum_{i=1}^{M_T}\sum_l |h_i[l]|^2]}{\sigma^2}.$$

Theoretical Analysis

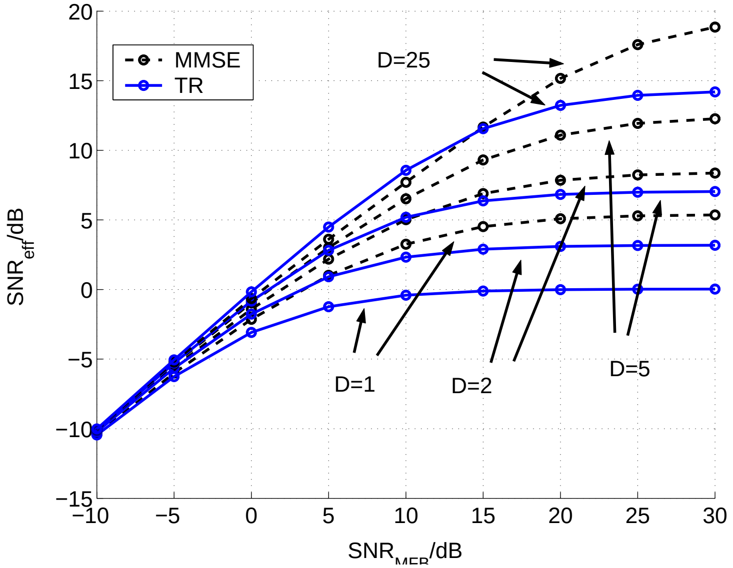
- The relationship between ρ_{eff} and ρ_{MFB} is

$$\rho_{\text{eff},D} = \rho_{\text{MFB}} \frac{1}{1 + \rho_{\text{MFB}} \frac{E[Q_{TR}^{[D]}]}{E[P_{TR}]}}.$$

- For an exponentially decaying PDP with delay spread σ_τ, we have derived limiting cases:

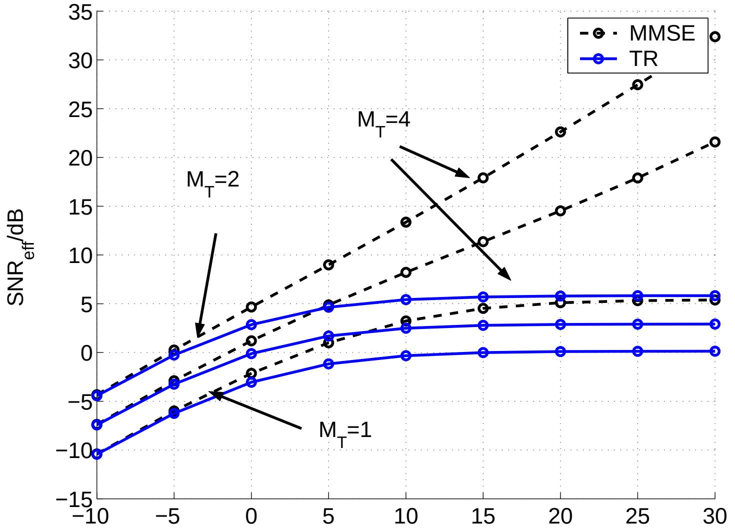
$$\begin{aligned} \rho_{\text{eff},D} &= \rho_{\text{MFB}} && \text{for } \sigma_\tau \longrightarrow 0 \\ \rho_{\text{eff},D} &= \rho_{\text{MFB}} && \text{for } \rho_{\text{MFB}} \text{ low, } \sigma_\tau \text{ large} \\ \rho_{\text{eff},D} &= DM_T && \text{for } \rho_{\text{MFB}},\ \sigma_\tau \text{ large.} \end{aligned}$$

Performance wrt Rate-back off D



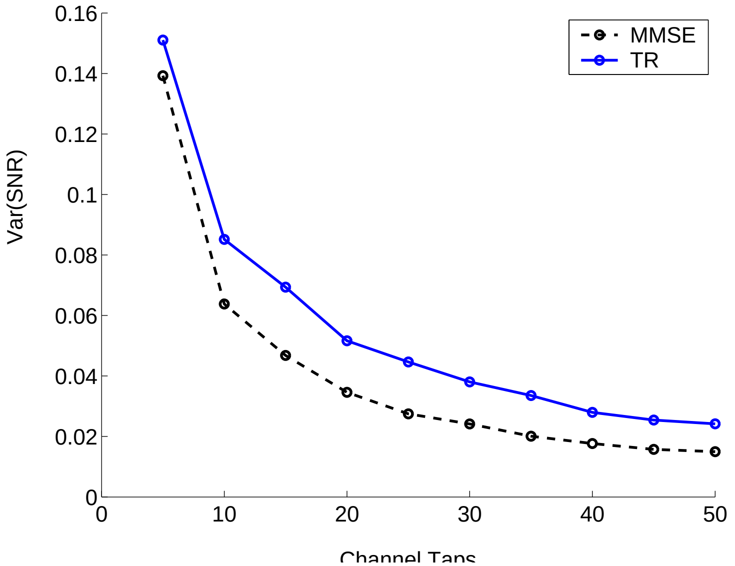
- We have compared TR with the optimal scheme.
- For TR ρ_{eff} saturates at high SNR to a particular limiting value.
- We increase this limiting value by increasing D.

Performance wrt Number of Tx Antennas



- We have compared TR with the optimal scheme.
- Doubling of N_T yields 3 dB more power for TR.
- The optimal scheme achieves nearly complete ISI cancellation for $M_T > 1$.

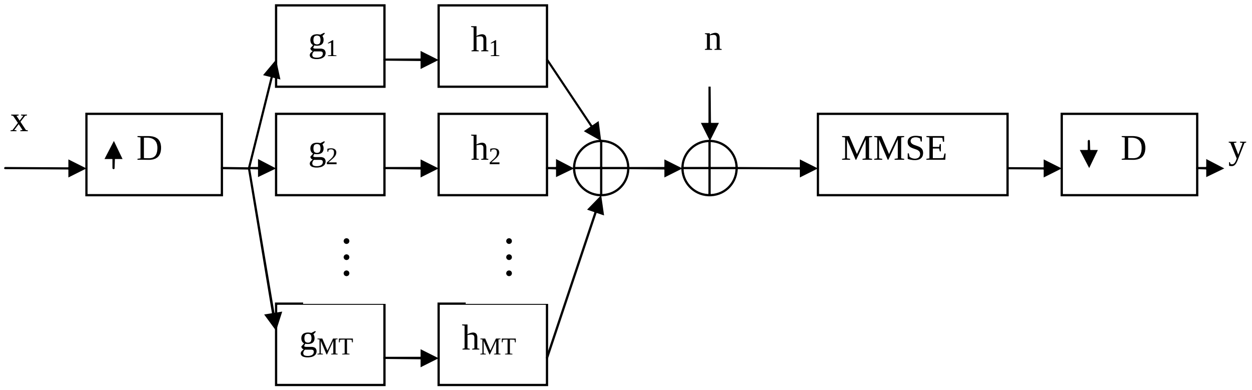
Channel Hardening



- Pre-equalization on long DS channels captures diversity.
- The variance of the received power decreases with increasing number of channel taps.

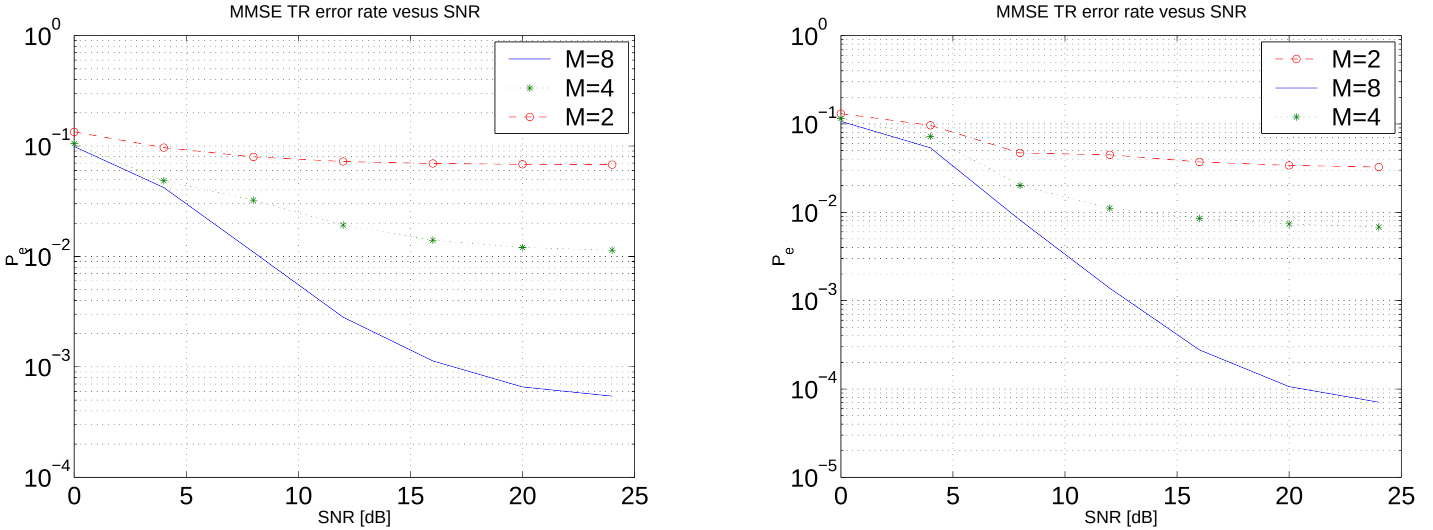
(to appear in Asilomar 2004)

Rate Back-Off and Receive Equalization - Intel UWB Data



- Combine TR with MMSE receive equalizer
- Compute bit error rates
- Investigate improvement with rate-back off
- Investigate improvement with increasing length of the MMSE filter

BER for a TR-UWB system with rate-back off



- Left: 10 tap MMSE receiver, right: 20 tap MMSE receiver
- The comparison is fair: the rate of all the schemes is the same

(to appear in Globecom 2004)

Rate back-off and optimal precoding - Summary

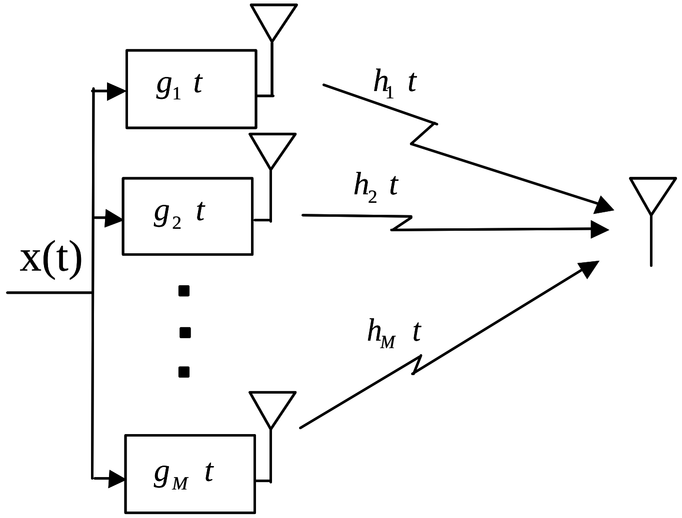
- There remains significant ISI at Rx after TR
- Rate back-off reduces ISI power linearly by approximately the same factor
- Optimal precoding can be used to reduce this residual ISI further

Channel Shortening with TR - Toronto Data

- We have investigated how the focusing works in case of real data.
- Here, we have used MMSE receive equalizer.
- We have chosen here that $\tilde{\gamma}_{TR} = \sigma_\tau^{(\mathrm{TR})} / \sigma_\tau^{(\mathrm{no\ TR})}$.
- We have investigated the impact of the number of the transmit antennas.
- We have investigated the improvement in transmission rate because of better SNR.

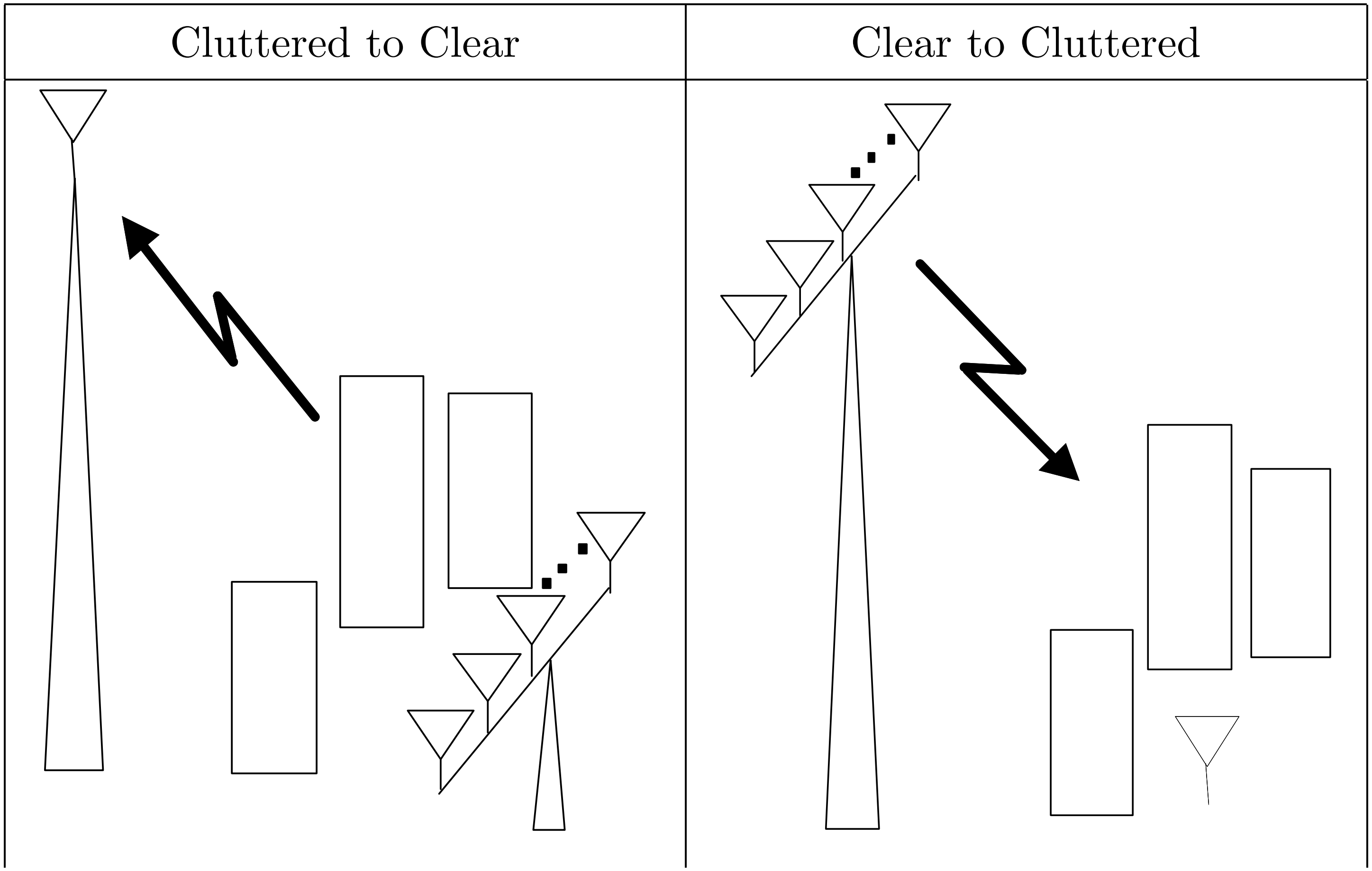
(Kyritsi et al, IEEE Antennas and Wireless Propagation Letters, 2004).

Toronto Data — Setup



- $g_m(t) = A_m h_m^*(-t)$
- $R_{h_m h_m}(t) = h_m^*(-t) \otimes h(t)$
- $h_{eq}(t) = \sum_{m=1}^{M} A_m R_{h_m h_m}(t)$
- For pure TR $A_m = \frac{1}{\sqrt{\sum_m \|h_m\|^2}}$
- Alternatively A_m can be chosen to minimize the delay spread

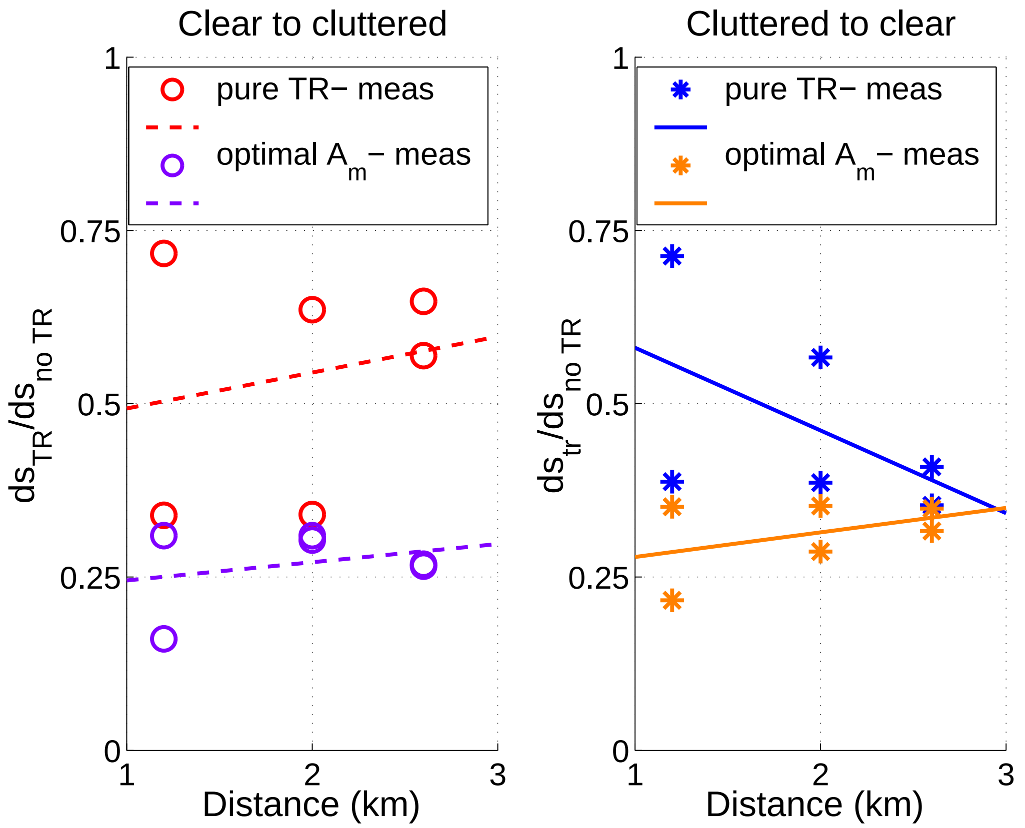
Scattering Scenarios



Cluttered to Clear

Clear to Cluttered

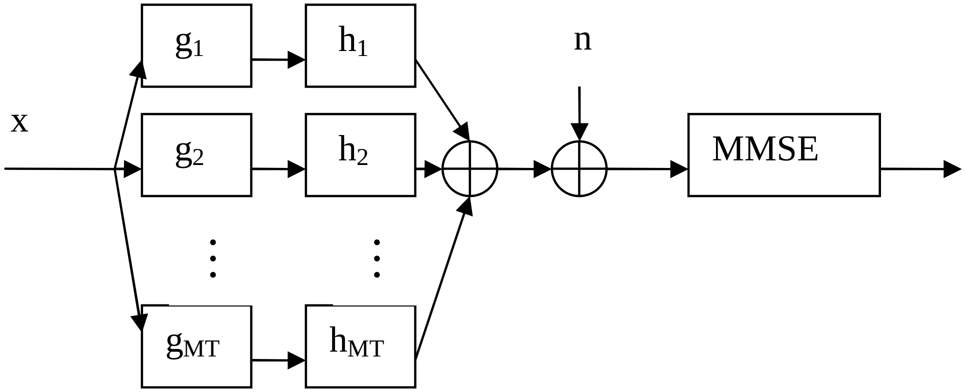
Channel Shortening Results



Comments on Channel Shortening

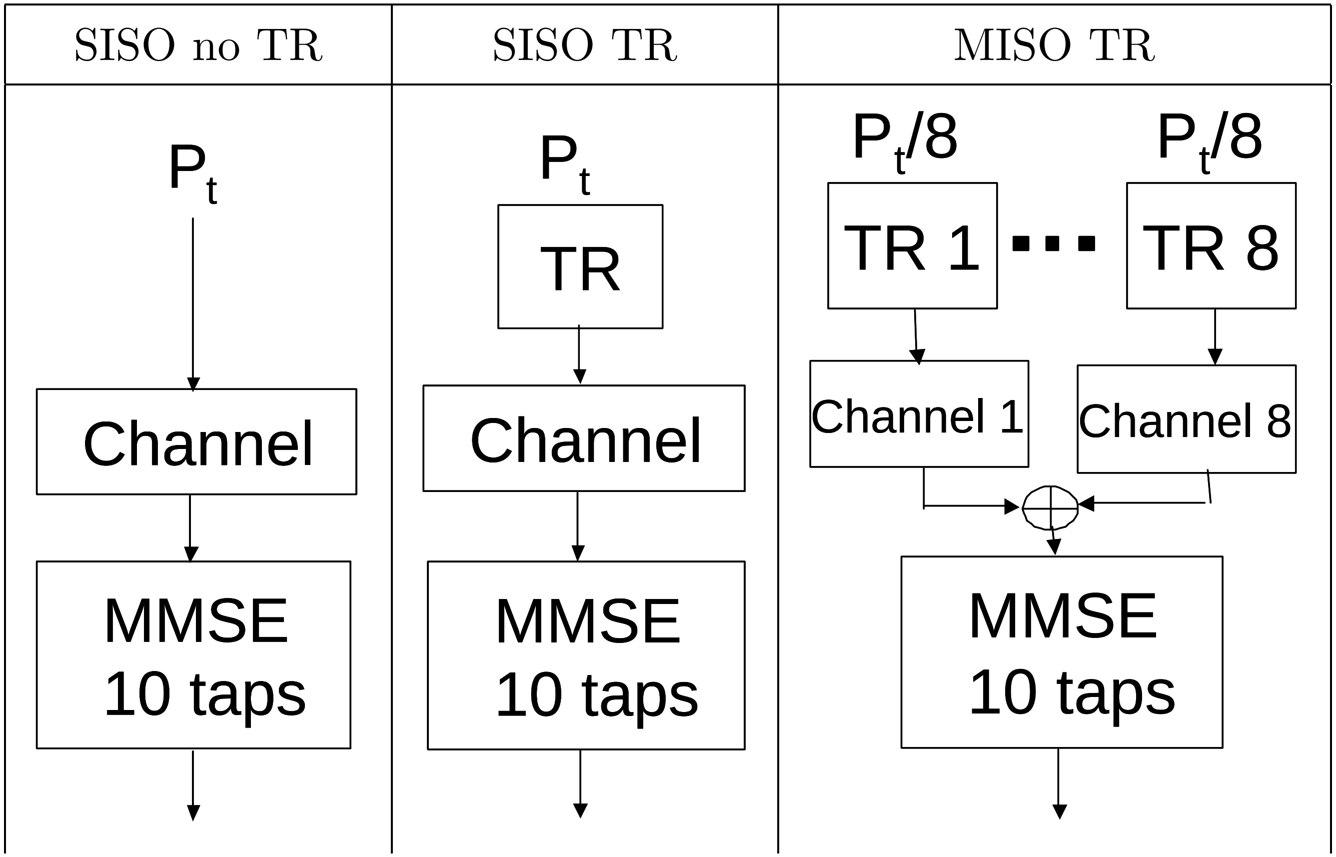
- More transmit antennas help reduce DS
- Pure TR can reduce the DS by a factor of 2
- Optimal TR can reduce the DS by a factor of 3
- Optimal TR performs independently of the channel characteristics (it makes transmission from clear to cluttered and from cluttered to clear equivalent).
- The efficiency of TR depends on channel characteristics

Rate Improvement for Fixed Complexity

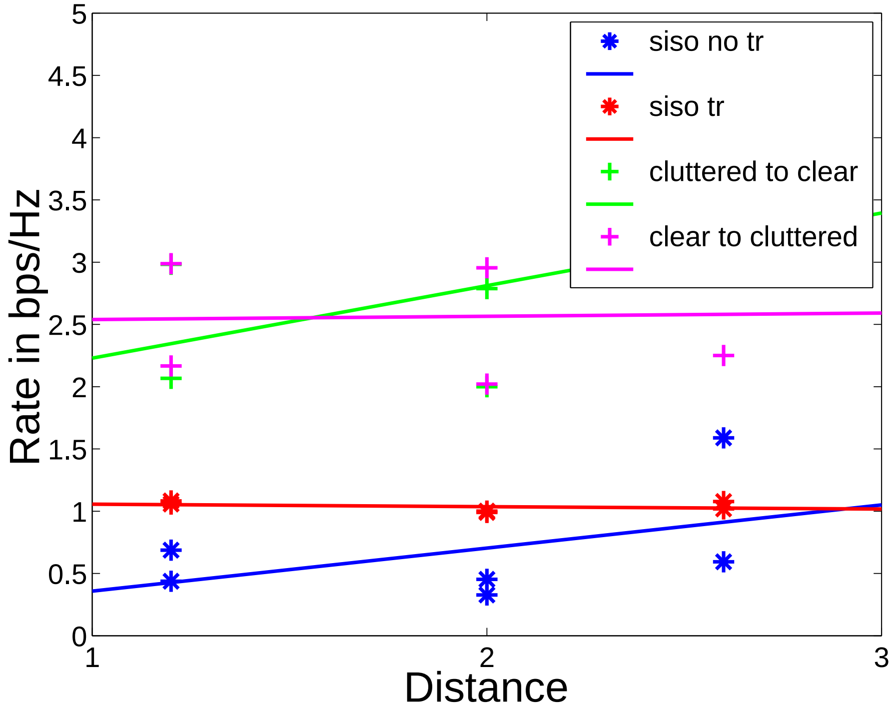


- System requirements
  - Constant transmit power
  - Target $BER = 10^{-3}$
  - Channel decoder: MMSE Rx with 10 taps
- Objective: What is the maximal rate that can be transmitted with and without TR precoding to satisfy the system requirements?

Comparison of TR and non-TR Schemes



Rate Improvement Results

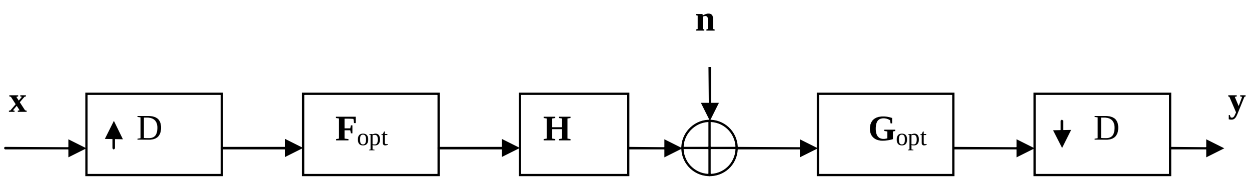


- More than x3 rate improvement with MISO TR

Work in Progress and Future work

- Comm systems with optimized precoding/decoding for receivers with only a few taps
- Analysis of the trade-offs implied by the optimization with respect to interference suppression and spatial focusing (preliminary results in this direction have been derived with the Toronto data and the numerical waveguide simulation data).

Joint Optimization of a Complex Tx and a Simple Rx

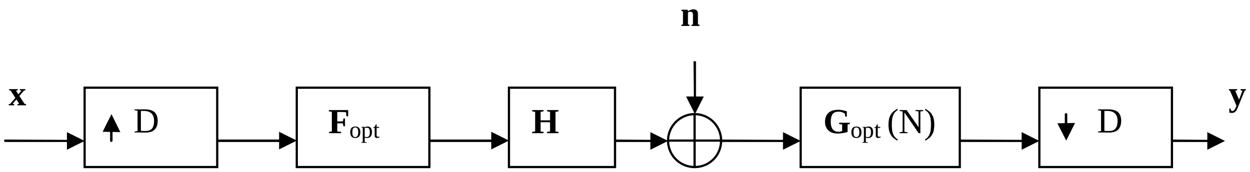


- For a system with transmit and receive filters, we optimize

$$\mathbf{y} = \mathbf{GHFx} + \mathbf{Gn}$$

- This has been done for **G**, **F** being matrices (Scaglione et al, 2002)
- For this scheme Tx does the power allocation, and Rx does the equalization

Joint Optimization of a Complex Tx and a Simple Rx (2)



- We perform this evaluation for long channels and under two restrictions:
  - The matrices **F** and **G** represent filters (i.e., they are Toeplitz matrices)
  - The receive filter **G** has length N and is very short!

Complex Transmitters and Simple Receivers (3)

- We minimize the mean square error MSE:

$$\begin{gathered}\min_{\mathbf{F},\mathbf{G}} E\|\mathbf{GHFx}-\mathbf{Gn}-\mathbf{x}\|^2\\ \text{s. t. trace}\{\mathbf{F}^H\mathbf{F}\}=1\\ \text{and diag}\{\mathbf{G}\}=\mathbf{M}^H\mathbf{Ta}\end{gathered}$$

$\mathbf{T}=\text{diag}\{[\mathbf{1}\quad\ldots\quad\mathbf{1}\quad\mathbf{0}\ldots\quad\mathbf{0}]\}$, $\mathbf{M}$: inverse Fourier transform matrix, $\mathbf{a}\in\mathbb{C}^N$

Complex Transmitters and Simple Receivers (4)

- We rewrite this optimization problem as

$$\begin{aligned} &\min_{\mathbf{F},\mathbf{G}} \mathbf{1}^T \left(\mathbf{H}^H\mathbf{F}^H\tilde{\mathbf{M}}^T\tilde{\mathbf{M}}^*\mathbf{F}\mathbf{H} + \bar{\eta}\mathbf{I}\right)^{-1}\mathbf{1} \\ \text{s. t.} \quad &\text{trace}\{\mathbf{F}^H\mathbf{F}\} = 1 \end{aligned}$$

- For a full length receive equalizer, the transmitter does only power allocation
- For a reduced length equalizer, the transmitter must take over some part of the channel shortening
- The solution is in progress.

Channel Estimation and Partial CSI

- In the training phase, the transmitter obtains a noisy estimate of the CSI with signal-to-noise ratio $\mathrm{SNR}_{\mathrm{Tx}}$.
- This is time reversed and amplified before transmission.
- The received signal has signal-to-noise ratio $\mathrm{SNR}_{\mathrm{Rx}}$ which includes both thermal noise at the receiver and the impact of the channel mis-estimation.

Total SNR gain vs Channel Estimate Quality

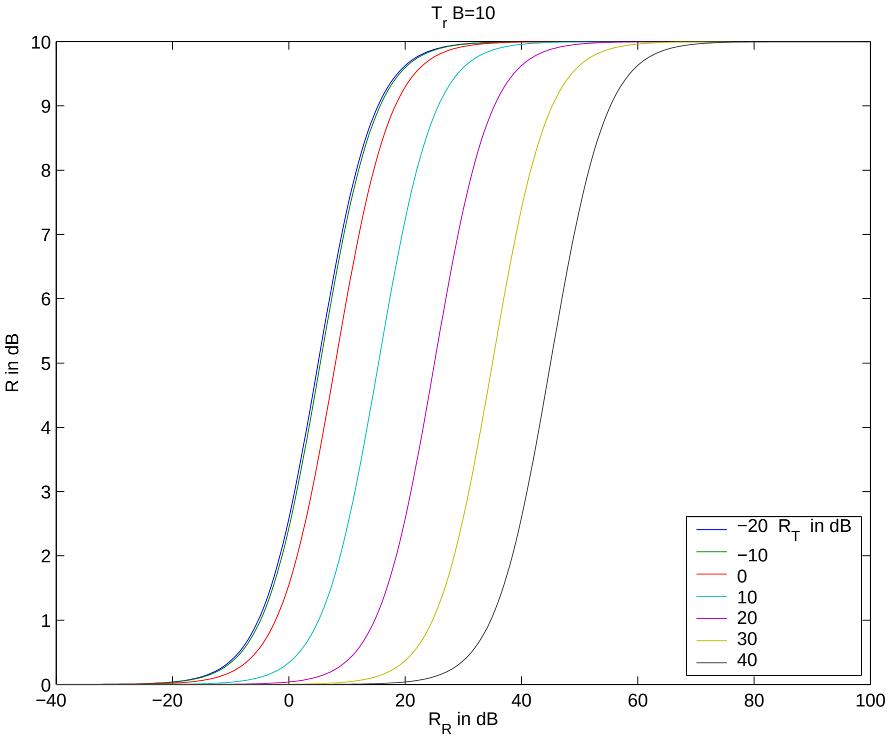
- Suppose the channel power to channel estimation error power is R_T
- the signal-to-noise ratio at the receiver is R_R
- the ratio of the received SNR using TR to that with Tx processing is

$$\frac{SNR_{\mathrm{TR}}}{SNR_{\mathrm{NONE}}} = \frac{1 + R_T + R_R}{1 + R_T + \frac{R_R}{BT_r}}$$

- BT_r is bandwidth time delay spread ($DS \times BW$)

(manuscript in preparation)

TR Performance vs CSI Quality at Tx



- When the channel estimate is accurate at the transmitter ($R_R >> 1$) the SNR gain for a TR system is approximately $DS \times BW$

(manuscript in preparation)

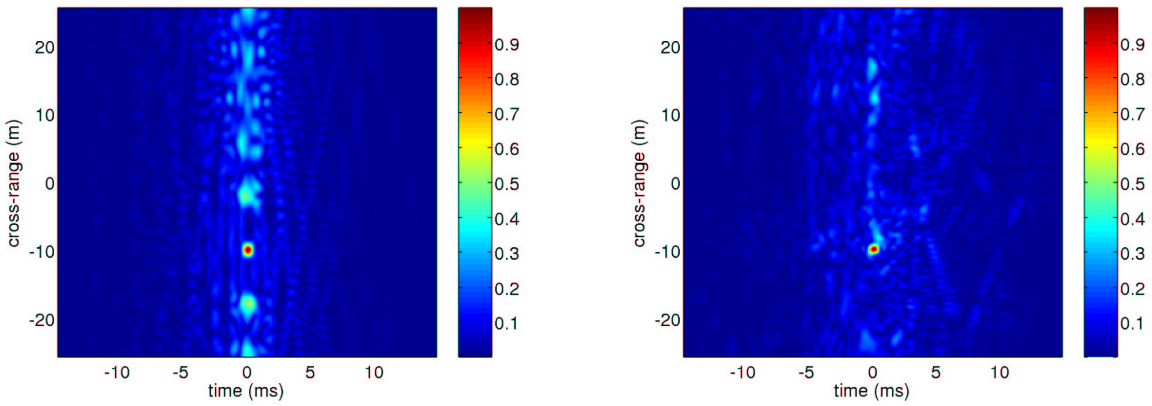
Robustness of TR against Channel State Error

- In TDD systems the transmitter obtains CSI on the reverse link
- In FDD systems CSI is fed back
- In both cases CSI is sensitive to channel variations in time
- TR is robust within limits against these channel variations

Spatial Focusing - Optimal Tx Processing

- TR suppresses interchannel interference
- We seek the optimal transmission scheme for a transmitter with high complexity, given:
  - a spatial channel model
  - instantaneous channel values to other users
- Nonetheless, the receiver may have to equalize the channel

Space-Time Focusing: Waveguide Simulations



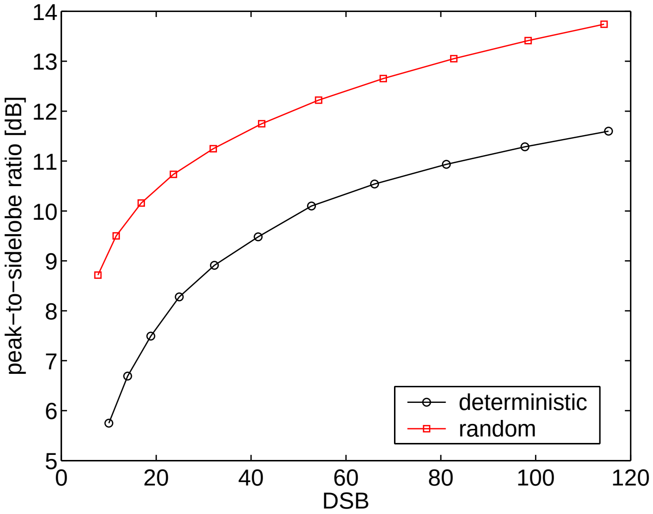
Space-time plots of time reversed signals

deterministic random

- Time reversal produces focusing in space and time.
- Random inhomogeneities add angular diversity.
- This additional diversity yields tighter focusing relative to propagation in the deterministic waveguide.

(to appear in IEEE J. Ocean. Eng., 2004)

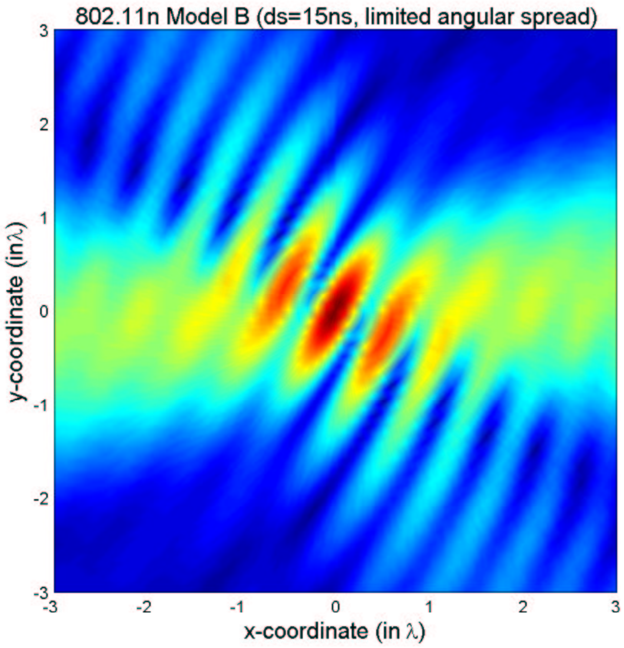
Spatial Focusing: Simulation Results

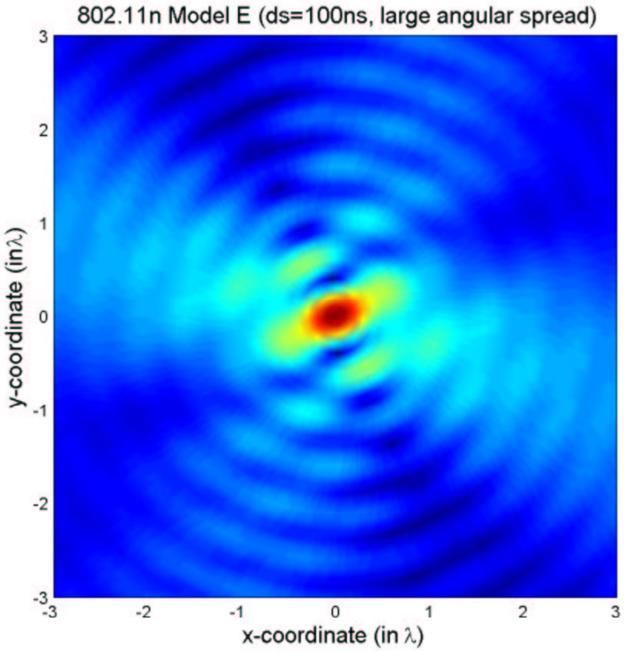


- We show the peak-to-sidelobe ratio as a function of the DSBW computed from simulations in the waveguide
- Spatial focusing is significantly better (~ 3 dB) with random inhomogeneities.

(to appear in IEEE J. Ocean. Eng., 2004)

Spatial Focusing in 802.11 channels


802.11n Model B (ds=15ns, limited angular spread)


802.11n Model E (ds=100ns, large angular spread)

- 802.11n channels have 10ns resolution and include directional information.
- We show the expected value of the field at $\tau = 0$ at off target locations (the target is at $(0, 0)$).
- The larger the delay and the angular spread, the tighter the focusing.

Spatial Focusing Example — Independent Networks

- Suppose two independent networks coexist
- We shall compute the optimal SISO BTS transmitter in terms of interference suppression towards the other network
- The channel to any other receiver $\mathbf{g}$ is independent of the channel between Tx and Rx, $\mathbf{h}$

Independent Networks (2)

- Assume full rate back-off
- The interference power I and desired power P are

$$\begin{aligned} I &= E[\mathbf{f}^H \mathbf{g}\mathbf{g}^H \mathbf{f} | \mathbf{h}] \\ P &= E[\mathbf{f}^H \mathbf{h}\mathbf{h}^H \mathbf{f} | \mathbf{h}] \end{aligned}$$

  f: transmit filter, h: channel in the TR network g: channel in the network suffering interference

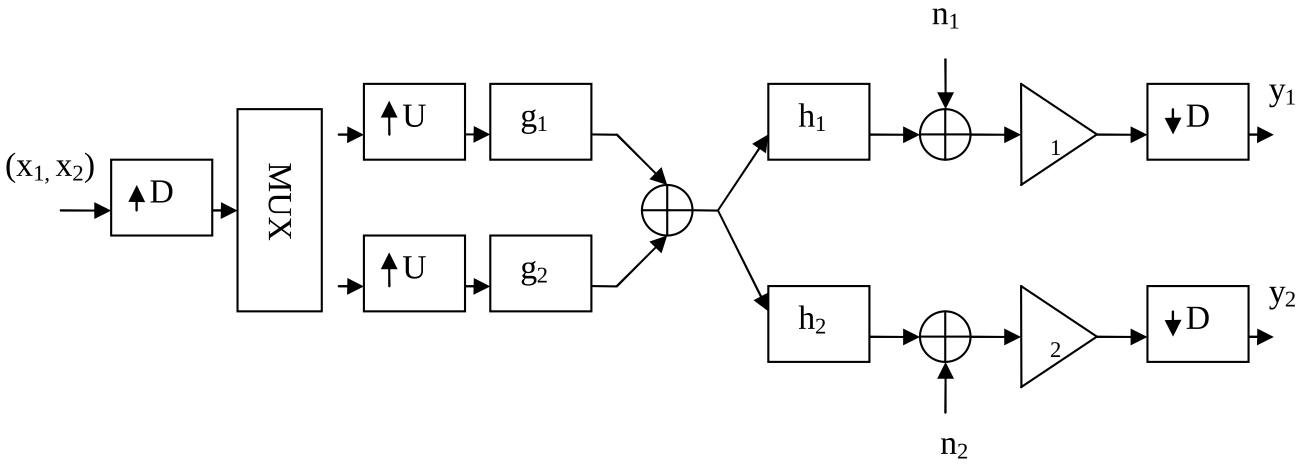
- When different tap coefficients are independent, the solution is TR:

$$\mathbf{f}_{opt} = \mathbf{h}^H$$

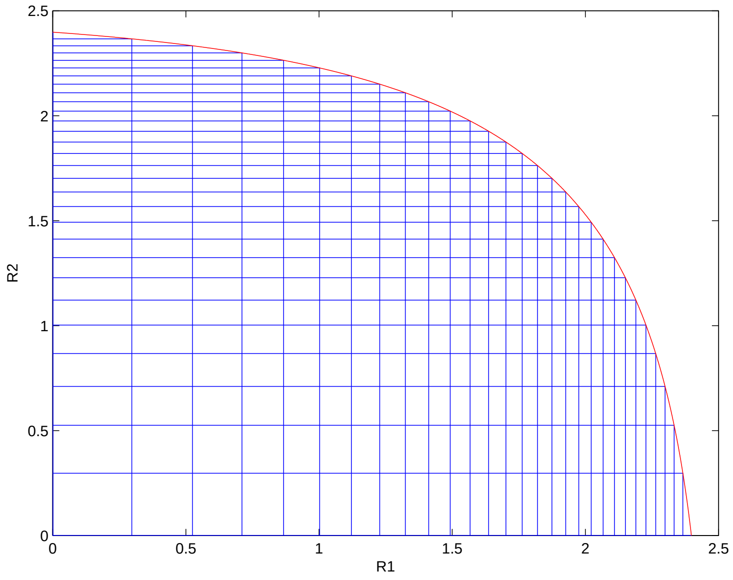
- The problem can be solved also when the tap coefficients are dependent

MU Communications

- Spatial focusing makes TR suitable for communications with several users

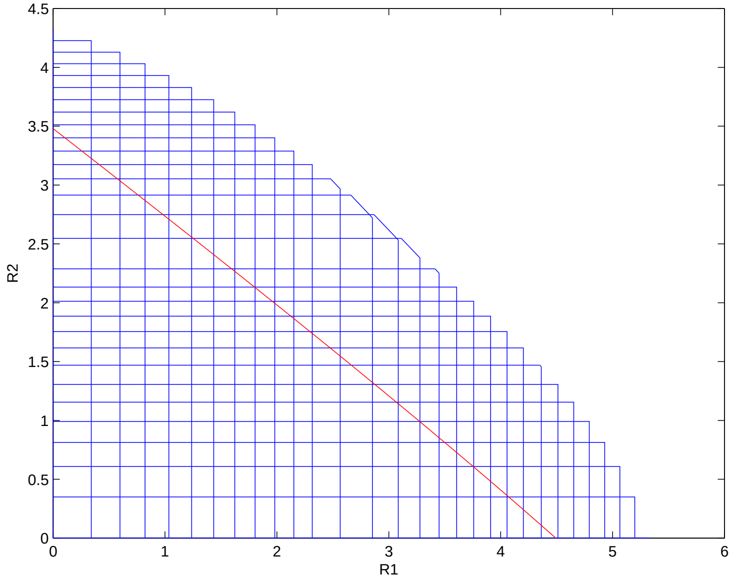


Two-User Rate Regions



- The achievable rate region of broadcast channel has been computed (Wei Yu, 2003)
- We have depicted here a two-tap orthogonal channel
- For this case TR achieves boundary of the rate region

Two-User Rate Regions (2)



- In reality, the former case is unlikely to happen
- Since no particular measures against interference are taken, TR BC rate is below time-sharing
- At low SNR TR is better, since the impact of the interference is less strong
- Rate-back off is also a good tool for MU-TR communications

Optimal Tx for MU with One-tap Rx

- We solve the following optimization problem

$$\begin{aligned} \min\ \max_{g_i,\beta_i} \quad & \left\{E\|\beta_1 \mathbf{x}_1^H \mathbf{H}_1 \mathbf{g}_1 + \beta_1 \mathbf{x}_2^H \mathbf{H}_1 \mathbf{g}_2 + \beta_1 n_1 - x_{1,\Delta}\|^2 ,\right. \\ & \left. E\|\beta_2 \mathbf{x}_2^H \mathbf{H}_2 \mathbf{g}_2 + \beta_2 \mathbf{x}_1^H \mathbf{H}_2 \mathbf{g}_1 + \beta_2 n_2 - x_{2,\Delta}\|^2\right\} \\ \text{s. t.} \quad & \mathbf{g}_1^H \mathbf{g}_1 + \mathbf{g}_2^H \mathbf{g}_2 \le 1 \end{aligned}$$

- The solution takes the form

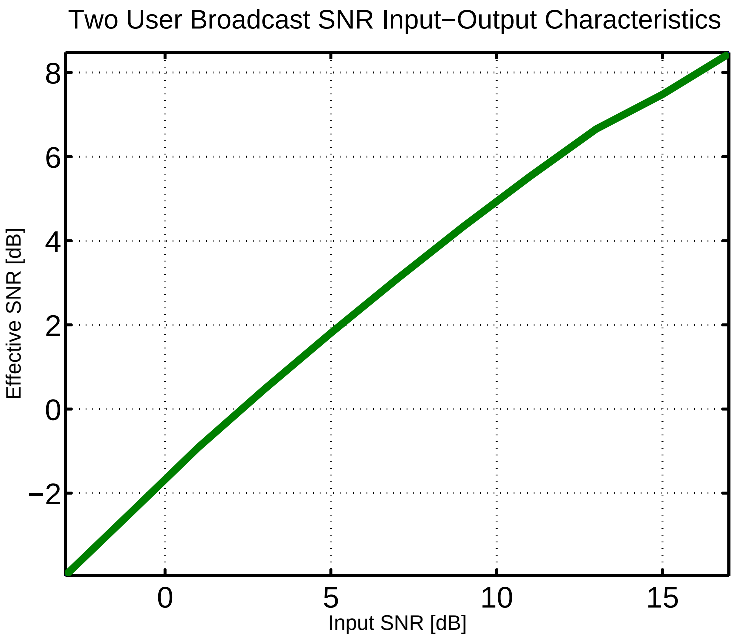
$$\mathbf{g}_i = \lambda_i P \beta_i \mathbf{Q}^{-1} \mathbf{H}_i^H \mathbf{e}_\Delta$$

where

$$\mathbf{Q} = \left(\lambda_1 P \beta_1^2 \left(\mathbf{H}_1^H \mathbf{H}_1 + \bar{\eta}\mathbf{I}\right) + \lambda_2 P \beta_2^2 \left(\mathbf{H}_2^H \mathbf{H}_2 + \bar{\eta}\mathbf{I}\right)\right)$$

λ_i: Lagrange multipliers

Performance Plot for Two-user System



- 2 single-tap receivers
- The rate-back off of 2
- The transmit prefilter 4 times longer than the channel

Conclusions

- Communications over large bandwidth offers potentially large data rates.
- TR, and improved schemes related to TR, allow data transmission in these channels with low complexity receivers
- The fundamental idea of these schemes is to shorten the channel by pre-equalization.
- Pre-equalizers can be combined with more sophisticated transmission (and precoding) schemes.
- Good spatial focusing properties enable large user density and operation in low probability of intercept situations.
- The trade-off between strong channel shortening and strong interference suppression is currently not understood fully.