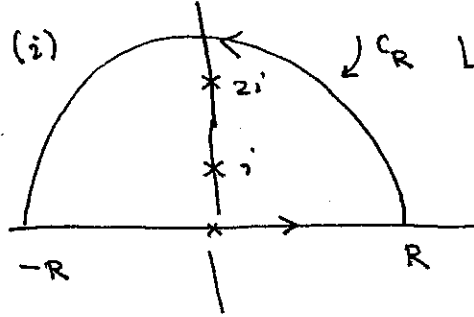


There's no $\frac{1}{z}$, but this doesn't mean that the residue of f at 0 is 0 because this expansion is valid too far away from 0 .

(iv). All terms appearing in (*) : $\frac{1}{z^3}, \frac{a+b}{z^4}, \frac{a^2+ab+b^2}{z^5} \dots$ integrate to 0 on any closed contour by F.T.C. (they have anti-derivatives) $\Rightarrow \int_C f = \boxed{0}$.

(v). If C only encloses the poles $z=0, z=a$, the answer becomes $2\pi i \left(\frac{1}{ab} + \frac{1}{a(a-b)} \right) = \boxed{2\pi i \frac{1}{b(a-b)}} \neq 0$.

(vi). By the same argument $\frac{1}{z-a_i} = \frac{1}{z} + \frac{a_i}{z^2} + \dots \Rightarrow \frac{1}{(z-a_1)\dots(z-a_n)} = \frac{1}{z^n} + \frac{a_1+\dots+a_n}{z^{n+1}} + \dots$ valid for $|z| > \max |a_i|$. If P has degree at most $n-2$, $\frac{P(z)}{(z-a_1)\dots(z-a_n)}$ has no powers of $\frac{1}{z}$ in its expansion so $\int_C \frac{P(z) dz}{(z-a_1)\dots(z-a_n)} = 0$.

II (i)  Let $C = C_R +$ the interval from $-R$ to R , with $R > 2$.

$$\int_C \frac{z^2 dz}{(z^2+1)(z^2+4)} = 2\pi i \left(\text{Res}_{z=i} + \text{Res}_{z=2i} \right)$$

$$\text{Res}_{z=i} \frac{z^2}{(z^2+1)(z^2+4)} = \frac{f(i)}{g'(i)} = \frac{i^2}{i^2+4} \cdot \frac{1}{2i} \text{ with } f = \frac{z^2}{z^2+1}, g = z^2+4$$

$$= -\frac{1}{6i}$$

$$\text{Res}_{z=2i} \frac{z^2}{(z^2+1)(z^2+4)} = \frac{(2i)^2}{(2i)^2+1} \cdot \frac{1}{2(2i)} = \frac{-4}{1-4} \cdot \frac{1}{4i} = \frac{1}{3i}$$

$$\Rightarrow \int_C \frac{z^2 dz}{(z^2+1)(z^2+4)} = 2\pi i \left(-\frac{1}{6i} + \frac{1}{3i} \right) = \boxed{\frac{\pi}{3}}$$

Now $\int_C = \int_{-R}^R + 2 \int_0^R \frac{x^2 dx}{(x^2+1)(x^2+4)}$

$$\left| \int_{C_R} \frac{z^2 dz}{(z^2+1)(z^2+4)} \right| \leq \int_{C_R} \frac{R^2}{(R^2-1)(R^2-4)} \cdot ds = \frac{R^2}{(R^2-1)(R^2-4)} \cdot 2\pi R \rightarrow 0 \text{ as } R \rightarrow \infty. \text{ Thus}$$

$$\boxed{\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{6}}$$

(ii). Use the same contour C as above for $\int_C \frac{e^{iaz}}{(z^2+b^2)^2} dz$. The residue at bi

$$\text{Res}_{z=bi} \frac{e^{iaz}}{(z^2+b^2)^2} = \text{Res}_{z=bi} \frac{F(z)}{(z-bi)^2} = F'(bi) \text{ with } F(z) = \frac{e^{iaz}}{(z+bi)^2}, F'(z) =$$