8 (i) $\operatorname{Res}_{z=\pi} \dfrac{z-\sin z}{z^2 \sin z} = \dfrac{f(\pi)}{g'(\pi)}$ with $f(z) = \dfrac{z-\sin z}{z^2}$, $f(\pi) = \frac{1}{\pi}$, $g = \sin z$, $f(\pi) = \frac{1}{\pi}$, $g'(\pi) = \cos\pi =$

Answer $\boxed{-\frac{1}{\pi}}$

(ii). $\operatorname{Res}_{z=-i} \dfrac{\sqrt{z}}{(z^2+1)^2} = \operatorname{Res}_{z=-i} \dfrac{f(z)}{(z+i)^2} = f'(-i)$ with $f(z) = \dfrac{\sqrt{z}}{(z-i)^2}$. Now

$$f'(z) = \frac{1}{(z-i)^4}\left\{\frac{1}{2}\cdot z^{-1/2}(z-i)^2 - z^{1/2}\cdot 2(z-i)\right\}.$$ For $z=-i$, $z^{1/2} = \left(e^{-\pi i/2}\right)^{1/2} = e^{-\pi i/4} = \dfrac{1-i}{\sqrt{2}}$.

$z^{-1/2} = e^{\pi i/4} = \dfrac{1+i}{\sqrt{2}}$. $\Longrightarrow f'(-i) = \dfrac{1}{(-2i)^4}\left(\dfrac{1}{2}\cdot\dfrac{1+i}{\sqrt{2}}(-2i)^2 - \dfrac{1-i}{\sqrt{2}}\cdot 2(-2i)\right) = \dfrac{1}{16}\Big(-\sqrt{2}(1+i) +$

$+ 2\sqrt{2}\,\underbrace{i(1-i)}_{i+1}\Big) = \boxed{\dfrac{1+i}{16}\cdot\sqrt{2}}$

9 $f(z) = \dfrac{1}{z+1} - \dfrac{1}{z+4}$. When $\left[\begin{array}{l} |z|<1,\ \dfrac{1}{1+z} = 1 - z + z^2 - z^3 + \dots \\ |z|>1,\ \dfrac{1}{1+z} = \dfrac{1}{z}\cdot\dfrac{1}{\frac{1}{z}+1} = +\dfrac{1}{z} - \dfrac{1}{z^2} + \dfrac{1}{z^3} + \dots \end{array}\right.$

When $\left[\begin{array}{l} |z|<4,\ \dfrac{1}{z+4} = \dfrac{1}{4}\cdot\dfrac{1}{\frac{z}{4}+1} = \dfrac{1}{4}\left(1 - \dfrac{z}{4} + \dfrac{z^2}{4^2} - \dots\right) \\ |z|>4,\ \dfrac{1}{z+4} = \dfrac{1}{z}\cdot\dfrac{1}{1+\frac{4}{z}} = \dfrac{1}{z}\left(1 - \dfrac{4}{z} + \dfrac{4^2}{z^2} - \dots\right). \end{array}\right.$

When $|z|<1$, $\boxed{f(z) = \left(1-\frac{1}{4}\right) - z\left(1-\frac{1}{4^2}\right) + z^2\left(1 - \frac{1}{4^3}\right) - \dots}$

$1<|z|<4$, $\boxed{f(z) = \left(+\frac{1}{z} - \frac{1}{z^2} + \frac{1}{z^3} + \dots\right) - \left(\frac{1}{4} - \frac{z}{4^2} + \frac{z^2}{4^3} - \dots\right)}$.

$|z|>4$, $\boxed{f(z) = \frac{3}{z^2} + \frac{1-4^2}{z^3} + \frac{-1+4^3}{z^4} + \dots}$

10 (i) Isolated singularities at $0, a, b$, all are poles.

$\operatorname{Res}_{z=0} f(z) = \dfrac{1}{ab}$, $\operatorname{Res}_{z=a} f(z) = \left.\dfrac{1}{z(z-b)}\right|_{z=a} = \dfrac{1}{a(a-b)}$, $\operatorname{Res}_{z=b} f(z) = \left.\dfrac{1}{z(z-a)}\right|_{z=b} = \dfrac{1}{b(b-a)}$.

(ii) $\displaystyle\int_C f(z)\,dz = 2\pi i\left(\frac{1}{ab} + \frac{1}{a(a-b)} + \frac{1}{b(b-a)}\right) = 2\pi i.\ \frac{(a-b)+b-a}{ab(a-b)} = \boxed{0}$.

(iii). $\dfrac{1}{z-a} = \dfrac{1}{z\left(1-\frac{a}{z}\right)} = \dfrac{1}{z}\left(1 + \dfrac{a}{z} + \dots\right)$

$\dfrac{1}{z-b} = \dfrac{1}{z\left(1-\frac{b}{z}\right)} = \dfrac{1}{z}\left(1 + \dfrac{b}{z} + \dots\right)$ $\Longrightarrow f(z) = \dfrac{1}{z^3}\left(1 + \dfrac{a}{z} + \dfrac{a^2}{z^2} + \dots\right)\left(1 + \dfrac{b}{z} + \dfrac{b^2}{z^2} + \dots\right)$

$\boxed{= \frac{1}{z^3} + \frac{a+b}{z^4} + \frac{a^2+ab+b^2}{z^5} + \dots}$