4 sides. Note $f = 2(x+iy) + (x^2-y^2+2ixy) = (2x+x^2-y^2) + i(2y+2xy) \Rightarrow$

$F = |f|^2 = (x^2-y^2+2x)^2 + 4(xy+y)^2$. We want max F.

- When $x=-1$, $F = (-y^2-1)^2 = \max \Leftrightarrow y^2 \max \Leftrightarrow y = \pm 1$. $\underline{F=4}$.
- When $x = 1$, $F = (3-y^2)^2 + 16y^2 = y^4 + 10y^2 + 9$. is increasing in y^2 so max

when $y^2 = 1 \Rightarrow F = 20$, $|f| = \sqrt{20}$, $\boxed{z = 1 \pm i}$

- When $y = \pm 1$, $F = (x^2+2x-1)^2 + 4(x+1)^2$., $F_x = 2(2x+2)(x^2+2x-1) + 8(x+1) =$

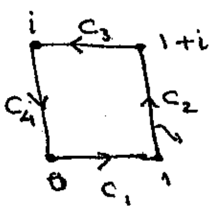
$= 4(x+1)(x^2+2x-1) + 8(x+1) = 4(x+1)(x^2+2x+1) = 8(x+1)^3 \geq 0$. Thus F is max again

when $x = 1 \Rightarrow \boxed{z = 1 \pm i}$ (again) and then $|f| = \sqrt{20}$.

6. (i). The pole $z=-1$ is not in $\mathcal{G}^+$ so c is holomorphic in $\mathcal{G}^+$. If $z \in \mathcal{G}^+$, write $z =$
$= x+iy$, $x>0$. We check $|c(z)|<1$ for $z \in \mathcal{G}^+$. this means $\left|\frac{z-1}{z+1}\right| < 1 \Leftrightarrow |z-1|^2 < |z+1|^2$
$\Leftrightarrow (x-1)^2 + y^2 < (x+1)^2 + y^2 \Leftrightarrow 4x > 0$ which is true.

(ii). If $f(z) \in \Delta$ for all z then $|f(z)| < 1$. Since f is entire, Liouville's thm implies f is constant. the answer is then $\boxed{\text{no}}$

(iii). If $f(z) \in \mathcal{G}^+$ for all z then $c(f(z))$ is well defined. and since $c: \mathcal{G}^+ \longrightarrow \Delta$ we have $c(f(z))$ is a holomorphic function with values in Δ so it must be a constant c. then $c = c(f(z)) = \frac{f(z)-1}{f(z)+1} \Leftrightarrow f(z) = -\frac{1+c}{1-c}$ also constant!! The answer is again $\boxed{\text{no}}$

7. 

$$\int_{C_1} \pi e^{\pi \bar{z}} dz = \int_0^1 \pi e^{\pi t} dt = e^{\pi t}\Big|_0^1 = e^{\pi} - 1.$$

$$\int_{C_2} \pi e^{\pi \bar{z}} dz = \int_0^1 \pi e^{\pi(1-it)} d(1+it) = -e^{\pi(1-it)}\Big|_0^1 =$$
$$= -e^{\pi(1-i)} + e^{\pi} = e^{\pi} + e^{\pi} = 2e^{\pi}.$$

$\underline{\text{on } C_3}$, $z = i + (1-t)$, $0 \le t \le 1$. $\int_{C_3} \pi e^{\pi \bar{z}} dz = \int_{C_3} \pi e^{\pi(1-t-i)}(-dt) = \cancel{\pi}\, e^{\pi(1-t-i)}\Big|_0^1 =$
$= e^{-\pi i} - e^{\pi(1-i)} = e^{\pi} - 1$.

$\underline{\text{On } C_4}$: $z = i(1-t)$, $0 \le t \le 1$. $\int_{C_4} \pi e^{\pi \bar{z}} dz = \int_0^1 \pi e^{+i(t-1)\pi} i(-dt) = -e^{i(t-1)\pi}\Big|_0^1 = -e^0 + e^{-i\pi} =$
$= -2$.

$\underline{\text{Answer}}$ $\boxed{\int_C \pi e^{\pi \bar{z}} dz = 4(e^{\pi} - 1)}$