Math 106 - Midterm 1.

The exam consists of 6 questions. Each page of the exam is worth 10 points. The maximum number of points is 50.

Name:

Acknowledgement and acceptance of honor code:

Signature:

Problem	Score
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Total	

Problem 1 (10 points)

Write the following complex numbers in standard form $a + bi$:

(i) [5]

$$\exp\left(-1 + \frac{i\pi}{4}\right)$$

(ii) [5]

$$\frac{1 + 3i}{1 - 2i}$$

Problem 2 (10 points)

(i) [5] Determine the domain of definition of the function

$$f(z) = \frac{z^2 + 1}{z^2 - 16i}.$$

(ii) [5] Compute the derivative of the function

$$f(z) = \frac{z - 1}{2z + 1}$$

at $z = -1 + \frac{i}{2}$.

Problem 3 (10 points)

True or false:

(i) [2] $\sqrt{z}$ is an entire function.

(ii) [2] the complex numbers z with $|z - 2 + 3i| = 1$ form a line in the complex plane.

(iii) [2] a function f is holomorphic if and only if $\frac{\partial f}{\partial \bar{z}} = 0$.

(iv) [2] $\lim_{z\to\infty} e^z = \infty$.

(v) [2] $u(x, y) = x^3 + y^3$ is harmonic.

Problem 4 (10 points)

Let $u(x, y) = x - y + 2x^2 - 2y^2$.

(i) [6] Check that u is harmonic. Find a holomorphic function f whose real part is $u(x, y)$ and $f(0) = 0$.

(ii) [4] Write f as a function of z alone.

Problem 5 (5 points)

Using the Cauchy-Riemann equations verify that the function

$$f(x, y) = 2x^2 - 2y^2 + 2xy + i(-x^2 + y^2 + 4xy)$$

is entire.

Problem 6 (5 points)

Determine the 9th derivative of the function

$$f(z) = \exp\left((-1 + i\sqrt{3})z\right)$$

at $z = 0$.