

# Mathematics 147, Spring 2006

## Differential Topology

### Syllabus

<http://math.stanford.edu/~munson/math147.html>

<b>Instructor</b>	Brian Munson	<b>Course Assistant</b>	Andres Angel
<b>Office</b>	382-H	<b>Office</b>	380-T
<b>Office Phone</b>	3-7829	<b>Office Hours</b>	
<b>Office Hours</b>	M 1-2:30 W 2:00-3:30		

**Text:** *Differential Topology* by Victor Guillemin & Alan Pollack.

### Course Description

Differential topology is the branch of topology that uses the tools of calculus (the “differential” in “differential topology”) to study topological spaces. The kind of topological spaces we study in differential topology are called manifolds. For now, think of a manifold as a surface or a higher-dimensional generalization of a surface. For example, the surface of a ball or a donut are examples of two-dimensional manifolds, and the solid ball and solid donut are examples of three-dimensional manifolds. Even simpler, a curve is a one-dimensional manifold and a point is a zero-dimensional manifold. The two most important problems in this field are that of distinguishing manifolds from one another (equivalently, being able to tell when two are the same), and finding the group of symmetries of a given manifold. These are related: Once I know two manifolds are the same, I can ask: In how many distinct ways are they the same? Don’t worry if this seems abstract (what the heck do I mean by “the same”?) or if you don’t understand what “group of symmetries” means; while these are not the topics of this course, I just want you to be aware that the most fundamental questions are still the most important, and the hardest to answer, and we still have only partial answers to these questions.

The main focus of our studies will be the way manifolds intersect with one another. Not only is this accessible (we very humbly begin only knowing calculus and some linear algebra), but it is intimately related to understanding the fundamental problems I mentioned above, something which there is neither room nor sufficient context to explain. We will begin by giving a precise definition of a manifold. From there we will use calculus to study the behavior of functions from one manifold to another. This will lead us to the concept of transversality (also known as “general position”), which is the most important geometric tool in studying the ways manifolds intersect one another. Using our intersection theory, we will prove some surprising and interesting theorems, whose surprise I will not ruin here, and whose interest will become apparent when we get there. To be briefly concrete, by the end of the course

we will be able to make some progress on the question of distinguishing one pair of linked circles in three-dimensional space from another pair via something called the linking number (which, by the way, is yet another problem to which we only have a partial answer!).

## **Your Grade**

**Homework:** 20%

**Midterm Exam,** Wednesday, May 10th: 30%

**Final Exam,** Monday, June 12th, 8:30-11:30 AM: 50%

## **Homework**

Homework is the most important work you will do in this course because it helps both illuminate and solidify the concepts we discuss in lecture. You should begin working on it as soon as possible, because it may take as many as a few days for you to see how to solve some of the problems. You may work together, but I encourage you to think hard about the problems before asking others (including me) for help. There are obvious benefits to figuring out these problems on your own. Sometimes a wrong idea is more important than the right one, and being stuck is incredibly good for you, believe it or not. The first and most important step in starting homework: memorize the definitions and try to understand them in plain language as much as possible: you need precision to write correct proofs, and intuition to discover an idea for a proof. Late homework is not accepted.

## **Exams**

The midterm exam will be held in the evening of May 10th, and you will have two hours to complete it. The final exam will be take-home, and it will be due when our scheduled final exam ends. You will have about a week to work on it.

## **Office Hours**

You should come to office hours prepared to do some hard thinking. If, for example, you are having trouble with a homework problem, then you should come prepared to show me what you have done and where you are getting stuck. If you're having trouble understanding a concept, then be prepared to explain it to me, even though you think you don't understand it. I run office hours by having you leading the session at the chalkboard. I spend enough time there already, and having me solve your homework for you is boring for me and no help to you.

## Important Dates

April 23rd: Add deadline

April 30th: Drop deadline

May 10th: Midterm Exam (evening, TBA)

June 12th: Final Exam: due 11:30 AM.