

Math 115, Fall 2003
Homework 1
Due Wednesday, October 1

1. Decide which of the following are statements:
 - (a) Every positive real number x has a square root.
 - (b) $3 + n + n^2$.
 - (c) $x^2 + x + 1 > 0$.
2. Identify the hypothesis and conclusion for the following statements:
 - (a) If XYZ is a right triangle, then it has no obtuse angles.
 - (b) For every integer n , $1 + 2 + \cdots + n = n(n + 1)/2$.

Prove directly (that is, don't use contrapositive, etc.) on problems 3-6.

3. Prove: If XYZ is an isosceles right triangle, then the hypotenuse is $\sqrt{2}$ times as long as one of its legs.
4. Prove: If x and y are nonnegative real numbers that satisfy $x + y = 0$, then $x = 0$ and $y = 0$.
5. Prove: If x and y are real numbers such that $x^2 + 6y^2 = 25$ and $y^2 + x = 3$, then $|y| = 2$.
6. Prove: If p and q are odd integers, then $p \cdot q$ is odd.
7. Write the negation of the following statements:
 - (a) If $x < 0$, then $x^2 > 0$.
 - (b) If p and q are odd integers, then $p \cdot q$ is odd.
8. Use proof by contraposition to prove: If p and q are odd integers, then $p \cdot q$ is odd.
9. Use proof by contraposition to prove: If p and q are integers such that $p \cdot q$ is even, then p is even or q is even.
10. Use proof by contradiction to prove: If $a > 0$, then $1/a > 0$.
11. Use proof by contradiction to prove: If a is a rational number and b is an irrational number, then $a + b$ is an irrational number.
12. Prove: If x and y are non-negative real numbers, then $x + y = 0$ if and only if $x = 0$ and $y = 0$.