THINKING THE CALCULUS WAY
HOMEWORK: DERIVATIVE FORMULAS

For THURSDAY, July 8:

Section 2.9: 5, 8, 10, 36, 47
Section 3.1: 3, 6, 7, 14, 15, 20, 25, 45

For FRIDAY, July 9:
You have seen by now that using the definition to compute the derivative can be tedious and even difficult. For this reason it is useful to have general formulas, which we ask you to derive. These are proofs, so treat your write-up as such! Remember: you know what the formulas should be, so if you get stuck, try working backwards from the answer.

1. **Linearity:** If f and g are differentiable functions and a and b are real numbers, then

$$\frac{d}{dx}(af(x) + bg(x)) = af'(x) + bg'(x).$$

   It might be easier for you to break this into several smaller statements, which are outlined below.

   (a) If f and g are differentiable functions, then $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$.
   (b) If f is a differentiable function and a is a real number, then $\frac{d}{dx}(af(x)) = af'(x)$.
   (c) Now write a proof of the original statement of linearity, using steps (a) and (b) above.
   (d) Explain why $\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$ using linearity.

2. **Product Rule:** If f and g are differentiable functions, then the function $f \cdot g$ is differentiable and $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$.

3. **Power Rule:** Use mathematical induction and the product rule to prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ if n is a positive integer. In fact, it actually true for any non-zero real number in place of n. We will cover the case of negative integers below after doing the quotient rule, and later we will prove it in the case of any real exponent when we talk about derivatives of exponentials and logarithms. To get you started, compute the derivative of $x^2 = x \cdot x$ using the power rule. Can you use this answer along with the product rule to help you compute the derivative of x^3?

4. **Quotient Rule:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

   Here's an outline:

   (a) First prove that $\frac{d}{dx}(x^{-1}) = -x^{-2}$ from the definition of the derivative.
   (b) Now use part (a) and the product and chain rules to prove the formula for the quotient rule.

5. **Power Rule Redux:** Prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ if n is a negative integer using the quotient rule (or the chain rule) and the version of the power rule you proved above. Then prove the power rule for n any rational number using the chain rule.