

MATH 108 – HOMEWORK #3

Due by 5pm, Wednesday, May 20.

Note: (*) = **required**, (**) = **optional**, (***) = **unsolved**.

- (1) (a) (*) Find a formula, with proof, for the number of 4-cycle subgraphs in the complete graph K_n .
(b) (*) Find a formula, with proof, for the number of Hamiltonian cycles in K_n .
- (2) (a) (*) How many factors does the number $2^6 3^{10} 5^{12}$ have, including one and itself?
(b) (*) Find a formula for the number of factors of a number $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$, where the p_i are distinct primes.
- (3) (**) Prove that

$$\sum_{m=0}^n \sum_{k=0}^m \binom{m}{k} \binom{n}{m} = 3^n.$$

Can you give both algebraic and conceptual proofs?

- (4) (*) How many subsets of the set $\{1, 2, \dots, 10\}$ contain at least one odd integer?
- (5) (*) There are four men and six women. Each man marries one of the women. In how many ways can this be done?
- (6) (*) In how many different ways can the letters of the word MISSISSIPPI be arranged if the four S's cannot appear consecutively?
- (7) (*) Prove that $\sum_{i=0}^n i \binom{n}{i} = n2^{n-1}$, in a “conceptual” or “bijective” way.
- (8) (*) If a bakery has five kinds of cookies, in how many ways can a dozen be chosen?

- (9) (a) (*) In how many ways can 10 red bricks and 4 white bricks be placed in a row, so that no two white bricks are adjacent?
- (b) (*) In how many ways can six girls and three boys be lined up so that no two boys are together?
- (10) (*) Estimate 100001^8 , without using a calculator, and showing your work. 10^{40} is too coarse a guess; your answer should be within 10^{30} of the truth.
- (11) (a) (*) Use Stirling's formula to estimate $\log_{10} 52!$. Your answer should be within ± 1 . This gives an order of magnitude estimate for $52!$.
- (b) (*) Let N = the total number of orderings every deck of 52 cards ever made has ever been in. Give an "order of magnitude physics" type argument to estimate N , and compare your answer to the first part. (If you are not sure what I mean, Google the Fermi question "How many piano tuners live in Chicago?" This question is meant to be fun, and you have some room to play.)
- (12) How many ways are there to put 15 balls in 4 boxes if
- (a) (*) the boxes and balls are distinguishable,
- (b) (*) the boxes are distinguishable, but the balls aren't,
- (c) (**) balls are distinguishable, but boxes aren't, and
- (d) (**) neither boxes nor balls are distinguishable.