

1 General philosophy

Education is more than simply providing students with job skills or even practical skills. Education is a principal part of the process by which society reproduces itself, transmitting scientific, social, cultural and other knowledge. Mathematics is a principal part of that knowledge, different in character from all other knowledge, with its own distinctive and impregnable standards of reasoning and proof; it is the pinnacle of thought and logic. Thus, mathematics education is a crucial component of a civilized society.

Mathematical thought is a combination of rigour, objectivity, creativity and certainty unknown to any other field; it is epistemologically privileged. Therefore, education in mathematics is a unique training in fearless, logical, creative critical thinking. It is the sort of education required in a democratic society, and also provides training essential to understanding contemporary society — mathematics provides a basic literacy in the modern world. And the higher one goes, the deeper the connections and the more profound truths one finds — truths that continually advance our knowledge.

Mathematics teaching is in many ways pleasurable and empowering for students and teacher alike. There is great pleasure and excitement in its learning and appreciation, both aesthetically and practically.

2 My experience

I have taught in the following roles; see my CV for further details:

- **Teaching assistant:** At Stanford I have been the teaching assistant for many undergraduate courses, including introductory calculus, linear algebra and multivariable calculus. This involves taking regular sections, holding regular office hours and grading exams.
- **Course assistant:** For many undergraduate courses, including functional analysis, group theory and complex analysis. The precise role varies from course to course, but includes holding office hours, grading homework and exams, writing homework solutions, and giving some lectures.
- **Stanford summer mathematics camp (SUMaC):** As a staff member, I met with students individually every day to discuss their experiences at the camp and explain mathematics from the intensive courses they were taking.
- **Mathematical Olympiad:** I was heavily involved in the training programme of the Australian Mathematical Olympiad Committee from 1999 to 2004, including serving as Deputy Leader of the Australian team at the International Mathematical Olympiad from 2002 to 2004. This involved a great deal of lecturing, explaining, problem-solving, leading problem sessions, organisation, as well as general advice and guidance.
- **Other:** For several years I spent several hours a week taking small classes of gifted students from high schools and challenging them with olympiad-style problems. I have also tutored students privately, usually at the high school level.

I look forward to teaching many classes in the future — including upper-level classes in my fields of interest. As for lower-level courses, I have enjoyed teaching calculus and linear algebra.

3 Specific issues and approach

There are specific issues I find important; below is my approach to them, philosophically and practically in the classroom.

- **Routine and repetition:** Mathematics is difficult for many, and learning it is often hard work. It may be true that sometimes, to internalise difficult techniques, repetitious exercises are necessary; but in my view they should be regarded as a necessary evil. Routine exercises in lower-level courses can be degrading to the spirit, as well as inhibiting creative critical thinking. The pianist's scales are rote but necessary for the development of a technique that can adequately perform something expressive

and worthwhile; a production line is quite different. I regard routine, regurgitation and repetition as acceptable if and only if the first analogy applies — as means to a higher end, as evils employed only where necessary.

- **Fulfilment of potentials:** If mathematical education is to see itself as part of the broad educational enterprise, then it must take seriously the goal of developing individuals to their full potential. Deep understanding of mathematical concepts can be illuminating, liberating, and empowering. An empowering teaching approach cannot be uniform when abilities vary widely. If some find a course easy, they should be challenged. If some find it too difficult, they should be encouraged and assisted.
- **Motivation and perspective:** As a result of the purity and rich content of mathematical theories, motivation is not always readily apparent. At higher levels, material may be esoteric and applications distant or non-existent. At lower levels, students may lose perspective, immersed in a sea of details and techniques. Moreover, when students are confronted with involved algorithms or proofs, it may be tempting to remain contented with understanding only each individual step, one at a time, without comprehending the whole; thus losing the overarching picture and motivation. Wherever possible, I seek to explain the motivation of results, why we are interested in them, and what they mean in perspective. This is crucial: the human mind understands and orders mathematics by placement into contexts and broad pictures. Even a vague understanding of motivation can make mathematics meaningful and useful — equally at the high school level, as in the most abstruse research.
- **The teacher's power:** The teacher carries a great deal of authority — authority made legitimate by possession of worthwhile knowledge, and by transmission of understanding. Mathematics possesses objective standards of truth to an extent unknown in other fields, strengthening the intellectual authority of the mathematics teacher — who has the power to declare the student wrong in an absolute sense. The teacher should therefore remain acutely aware that the slightest condescension, impatience, or declaring a student “wrong”, can have a destructive impact on confidence, and a chilling effect on critical thinking. The goal must be to encourage independent thoughts and creative intellectual activities; this includes welcoming difficult questions and alternative approaches to problems, and encouraging discussion.
- **Fun and excitement:** Perhaps because of its privileged epistemological status — and also its difficulty — there is a special pleasure in comprehending mathematical concepts or solving a challenging problem. The obverse of this phenomenon is the frustration of non-understanding or non-solving; and the excitement of others can be as humiliating as it can be infectious. This narrow line between vitalizing or depressing students is common to all education — but I think especially in mathematics. Anything the teacher does potentially demands too much, too little, bores more capable students, or alienates less capable students. The teacher walks a fine line!

In the classroom I find myself constitutionally acutely sensitive to such possibilities. Students are often unwilling to report difficulties, for peer-related, pride or classroom-political reasons. I find generosity of spirit, fearless (but non-patronizing) intellectual compassion, and the resolute inability to regard any statement as obvious or any question as stupid, to be essential counterweights. I aim for an honestly enthusiastic (wherever possible!), bright classroom demeanour punctuated with occasional humour, careful planning, crystal clear explanations, and enjoying myself (I am usually having fun while explaining mathematics) — without lapsing into self-indulgent disregard for the feelings of students.

- **Basics:** Be punctual; come prepared; have a plan for class; make eye contact; understand the material; make clear what is expected from students and what they can expect from their instructor; empathize with students' difficulties; always be polite; speak clearly and at a measured pace; use good blackboard technique, writing down at least the main points of any discussion. Some things in teaching, as in life, are very simple, and getting the simple things right makes everything easier.

This is an ideal, and I am sure I do not always live up to it; educational philosophy and its practical applications are, for any active teacher, a work continually in progress.