

Contact topology of 3-manifolds with boundary

1 Introduction

Contact geometry, the odd-dimensional sibling of symplectic geometry, arises classically from the study of optics and classical mechanics. Despite its classical origin, the subject has exploded in the last two decades, including the development of powerful invariants for contact 3-manifolds. A contact structure on a 3-manifold is a non-integrable 2-plane field, but the methods of contemporary contact geometry go far beyond differential geometry and include methods which are highly combinatorial, as well as methods which are highly analytic.

One of my interests is the interplay between these combinatorial and analytic methods. The combinatorial aspect of 3-dimensional contact topology includes Giroux’s theory of convex surfaces [8], which reduces contact geometry near an embedded surface to the study of finite sets of curves on the surface known as *dividing sets*. It also includes Giroux’s theorem on contact structures and open book decompositions [6, 10]. The analytic aspect involves the study of pseudo-holomorphic curves, originally introduced by Gromov [11], and leading to theories of contact homology and symplectic field theory [4].

Another theme of my research has been to consider contact topological properties particularly of 3-manifolds *with boundary*. An important research programme in contact topology is to extend contact homology theories to 3-manifolds with boundary, in analogy with recent developments in Heegaard Floer homology theory.

2 Contact elements in sutured Floer homology

For present purposes, a *sutured 3-manifold* can be thought of as a 3-manifold with boundary, with some disjoint simple closed curves (annular sutures) drawn on the boundary. *Sutured Floer homology theory* [18] associates to certain sutured 3-manifolds (M, Γ) an abelian group $SFH(M, \Gamma)$; we will ignore signs throughout and consider it a \mathbb{Z}_2 -vector space; it is a generalization of the \widehat{HF} version of Ozsváth–Szabó Heegaard Floer homology theory [24, 25]. Heegaard Floer theory arises (in Lipshitz’s cylindrical reformulation [20]) from counting certain pseudo-holomorphic curves in $\Sigma \times I \times \mathbb{R}$, where Σ is a Heegaard surface. In sutured Floer homology, the notion of Heegaard decomposition is broadened to sutured 3-manifolds.

Sutured manifold theory is closely related to contact geometry via *convex decomposition theory* [14]. We can regard sutures Γ as providing a boundary condition for a contact structure: the boundary should be a convex surface and Γ should be the dividing set. Moreover, any contact structure ξ on a closed 3-manifold (M, ξ) determines a *contact element* $c(\xi) \in \widehat{HF}(M, \xi)$ [16, 26]; and similarly in the sutured case, a contact structure ξ satisfying our boundary condition determines a contact element $c(\xi) \in SFH(M, \Gamma)$ [15]. If ξ is overtwisted (resp. Stein fillable) then $c(\xi)$ is zero (resp. nonzero). Honda–Kazez–Matić [17] identified TQFT-like properties of SFH and contact elements.

A natural and important question is how contact elements lie inside $SFH(M, \Gamma)$. I have obtained some results, still partly conjectural (but computations leave little doubt), in the case where M is the solid torus $T = D^2 \times S^1$ and $\Gamma = \Gamma_n$ consists of $2n$ longitudinal sutures of the form $\{\cdot\} \times S^1$. The techniques used to obtain these results are likely to generalize to more involved cases and to elucidate the relationship between contact geometry and sutured Floer homology. Moreover, the conjectural answer is highly combinatorial; the conjecture which remains consists purely of some involved but elementary combinatorics. The result also gives an interesting analogy with (non-topological!) quantum field theory.

The \mathbb{Z}_2 -vector spaces $SFH(T, \Gamma_n)$ are easily computed from results of Juhasz [19] and Honda–Kazez–Matić [17]: $SFH(T, \Gamma_n) \cong \mathbb{Z}_2^{2^{n-1}}$.

The gluing theorem of Honda [13] implies that tight contact structures (up to isotopy) on (T, Γ_n) are in bijective correspondence with *chord diagrams* of n arcs. Given a circle with $2n$ distinguished points, a chord diagram is a collection of disjoint arcs joining these points in pairs. These chord diagrams represent the dividing set on a meridional disc of T ; the content of the gluing theorem is that all such diagrams define distinct tight contact structures. Thus the number of contact structures is C_n , the n ’th *Catalan number*. The $2n$ points divide the circle into $2n$ intervals which we alternately label positive and negative, and a chord diagram cuts the disc into positive and negative regions. The *relative euler class* e of the contact structure is given by the signed sum of these regions. The vector spaces $SFH(T, \Gamma_n)$ split as a direct sum (over spin-c structures) like Pascal’s triangle $\mathbb{Z}_2^{2^{n-1}} = \bigoplus_k \mathbb{Z}_2^{\binom{n-1}{k}}$, which we can also write as $\bigoplus_e SFH(T, \Gamma_n, e)$; and if ξ is a tight contact structure with relative euler class e , then $c(\xi)$ lies in the summand $SFH(T, \Gamma_n, e)$.

We can relate the various $SFH(T, \Gamma_n, e)$, in effect “categorifying Pascal’s triangle”.

Proposition 2.1 *There are “creation” and “annihilation” operators*

$$B_{\pm} : SFH(T, \Gamma_n, e) \longrightarrow SFH(T, \Gamma_{n+1}, e \pm 1), \quad A_{\pm} : SFH(T, \Gamma_n, e) \longrightarrow SFH(T, \Gamma_{n-1}, e \pm 1)$$

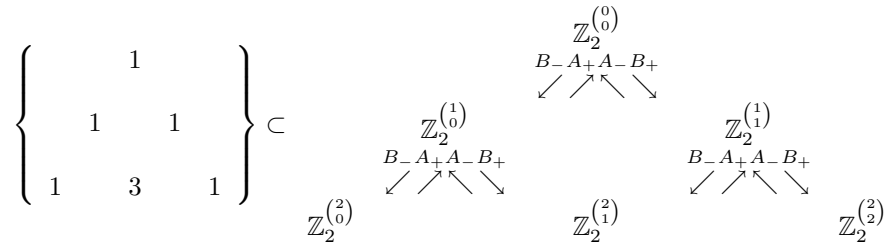
arising from inserting or conjoining arcs, which take contact elements to contact elements, and which satisfy:

$$A_+ \circ B_- = A_- \circ B_+ = 1, \quad A_+ \circ B_+ = A_- \circ B_- = 0.$$

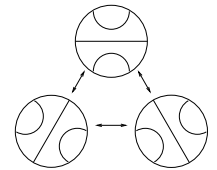
Moreover the creation operators are injective and “categorify the Pascal recursion”:

$$SFH(T, \Gamma_n, e) = B_+ (SFH(T, \Gamma_{n-1}, e - 1)) \oplus B_- (SFH(T, \Gamma_{n-1}, e + 1)).$$

The number of contact elements in each row of “Pascal’s categorified triangle” is given by C_n . Moreover we can denote by $C_{n,e}$ the number with euler class e , so $C_n = \sum_e C_{n,e}$. The $C_{n,e}$ form a “Catalan triangle” which obeys a simple recursion and describes the number of contact elements in the corresponding sutured Floer homology groups.



We can ask whether the vector space addition operation has any contact-geometric meaning. As it turns out, the answer involves *bypass moves*, which are surgeries on dividing sets introduced by Honda [12]. A bypass move takes an arc intersecting the chords in our diagram in precisely 3 points and performs either of the two surgeries shown. Dividing sets related by bypasses naturally come in triples as shown. One direction of the following result seems to have been known in [17].



Proposition 2.2 *Suppose $a, b, \in SFH(T, \Gamma_n, e)$ are contact elements. Then $a+b$ is a contact element if and only if the chord diagrams for a and b are related by a bypass move; and $a+b$ is the third diagram in their triple.*

The nonzero element $v_0 \in SFH(T, \Gamma_1, 0) \cong \mathbb{Z}_2^{(0)}$ is a contact element (“the vacuum”); applying B_{\pm} repeatedly to it, we obtain various contact elements. Note that the number of ways in which B_{\pm} can be applied to v to yield a result in $\mathbb{Z}_2^{(k)}$ is precisely $\binom{n}{k}$. Thus we have $\binom{n}{k}$ distinguished elements in $\mathbb{Z}_2^{(k)}$, which we can denote by v_w , where w is a word in the symbols $+$ and $-$, of length n and sum e . Such words are totally ordered lexicographically. They are also partially ordered (a sub-order of the lexicographic order) by the relation \preceq , “all minus signs move right”.

We now have the following result/conjecture about the location of contact elements, the first part of which is highly analogous to quantum field theory.

Theorem 2.3 (See [22])

- (i) *The v_w form a basis for $SFH(T, \Gamma_n, e)$. That is, “the space of all states has a basis consisting of the result of applying creation operators to the vacuum”.*
- (ii) *The number of pairs (v_{w_1}, v_{w_2}) of basis elements in $SFH(T, \Gamma_n, e)$ such that $w_1 \preceq w_2$ is precisely $C_{n,e}$.*
- (iii) *Writing a contact element as a sum of basis vectors, there is a lexicographically first and last basis vector amongst them. Two distinct contact elements give a distinct pair of such basis vectors. Furthermore, this pair of basis vectors is comparable under our partial order. Thus, we have a bijective correspondence between contact elements and comparable pairs of basis vectors.*

Various techniques can be used to study the problem; essentially, pictorial operations on chord diagrams give rise to linear algebra operations on \mathbb{Z}_2 -vector spaces. For instance, rotating a chord diagram gives rise to a “rotation operator” R which obeys a recursive formula:

$$R = \sum_{n=0}^{\infty} [B_+ R A_+, B_-^{n+1}] A_- A_+^n.$$

This allows us to compute R , and then to compute inductively the set of contact elements: any chord diagram can be rotated so that it has an outermost arc in a specified position, and hence by applying R sufficiently many times, any contact element in $SFH(T, \Gamma_n, e)$ can be taken to one in $B_{\pm}(SFH(T, \Gamma_{n-1}, e \mp 1))$. Using this computation, the conjectural part of the above result has been verified with high confidence.

One imagines this is the beginning of a combinatorial theory of contact elements in sutured Floer homology, with analogies to TQFT and QFT. It's clear that the creation and annihilation operators discussed here exist not just for meridional discs of solid tori, but in considering a general surface times S^1 ; so that there is the hope of computation or even solution of the (1+1)-dimensional TQFT described in [17].

There are clear next steps also in other directions: to extend to general slopes of sutures, not just the “vertical” ones considered above, where the number of contact structures is described by continued fractions [12]; and to other sutured 3-manifolds. Moreover, there are likely to be many other operations on SFH of interest and significance — in addition to “creation”, “annihilation” and “rotation”, there is a “merging” operation which categorifies the Catalan recursion, there are various types of “gluing”, and higher genus operations.

3 Contact homology for 3-manifolds with boundary

Contact homology theory is an invariant of a contact manifold obtained via counting pseudoholomorphic curves in its symplectization; it comes in various flavours, counting curves of various types. For a *closed* contact manifold (M, ξ) , we choose a contact form α (which determines a Reeb vector field R) and consider the symplectization $(M \times \mathbb{R}, \omega = d(e^t \alpha))$. We choose an almost complex structure J compatible with ω and consider J -holomorphic curves which are asymptotic to closed Reeb orbits at $M \times \{\pm\infty\}$ (“ \pm infinity”). One defines a chain complex generated by certain Reeb orbits and a differential to count certain J -holomorphic curves between them; the structure of the moduli space implies that $d^2 = 0$. One thus obtains the *contact homology* of (M, ξ) and proves that, under certain assumptions, it is independent of the choice of α and J [1, 2, 4].

One would like to extend contact homology theory in 3 dimensions to the case of a manifold with boundary. There are several approaches; we consider them in turn.

3.1 A transverse knot invariant

As a first case, we consider a transverse knot K in a contact manifold (M, ξ) . We note that there exist choices of contact form α for which K is a Reeb orbit. In fact, for any $c > 0$, there exists a solid torus neighbourhood $D^2 \times S^1 = (\{0\} \times S^1) \cup (((0, 1) \times S^1) \times S^1)$ in which every torus $(\{\cdot\} \times S^1) \times S^1$ is foliated by Reeb trajectories of the same slope c (where $c = 0$ is a meridian). If c is irrational, then there are no other Reeb orbits near K .

We take a chain complex generated by Reeb orbits away from K , and a differential d counting rigid J -holomorphic curves in $M \times \mathbb{R}$ between them, as usual. Note that $K \times \mathbb{R} \subset M \times \mathbb{R}$ is a J -holomorphic curve. There is a well-defined intersection number between this curve, and any J -holomorphic curve counted in d . Hence we may adjoin a formal variable t to our chain complex and define our differential as

$$d = d_0 + d_1 t + d_2 \frac{t^2}{2!} + \dots$$

where d_i counts those curves with i intersections, thereby filtering our contact homology by intersections with $K \times \mathbb{R}$.

To show that $d^2 = 0$, we must ensure that the codimension-1 boundary of our moduli space contains only curves counted in our chain complex. In particular, we must ensure that no curve approaches $K \times \mathbb{R}$ as a limit. We can carry this out by first choosing α such that the Reeb slope c is irrational and gives K an index of N ; then we consider a chain complex generated by Reeb orbits with grading $\leq N - 1$; so that, with generic J , K cannot arise as a Reeb asymptote in any codimension-1 limit. The homology so obtained is independent of the choice of α obeying such conditions; and taking $N \rightarrow \infty$ then gives a homology theory $CH^-(M, \xi, K)$.

Note that there is also the simpler theory $\widehat{CH}(M, \xi, K)$ obtained by considering $d = d_0$, i.e. curves which avoid $K \times \mathbb{R}$. (We use the notation CH^- in analogy with HF^- from Heegaard Floer theory, which keeps track of intersections with a basepoint; and \widehat{CH} in analogy with \widehat{HF} , which counts curves avoiding the basepoint.)

Theorem 3.1 *Let (M, ξ) be a closed contact manifold and K a transverse knot. One can define a “transverse knot contact homology” $CH^-(M, \xi, K)$ which is a $\mathbb{Z}_2[t]$ -module invariant of (M, ξ, K) . One can also define $\widehat{CH}(M, \xi, K)$, which is a \mathbb{Z}_2 -vector space invariant.*

Question 3.2 *How can $CH^-(M, \xi, K)$ and $\widehat{CH}(M, \xi, K)$ be computed?*

We remark that the above immediately applies to Legendrian knots, by considering a (positive or negative) transverse pushoff K_+ or K_- ; Legendrian knots related by stabilization will have the same transverse pushoff, and hence the same associated module.

In the case of a Legendrian knot K , we might be able to go further: $K \times \mathbb{R}$ is Lagrangian; and we might not only consider a homology filtered by intersections with $K_{\pm} \times \mathbb{R}$, but also allow Lagrangian boundary conditions along $K \times \mathbb{R}$. This should give rise to a generalized version of *Legendrian contact homology* [3, 7, 23] which counts such curves.

3.2 Pre-lagrangian torus boundary

The above is actually a special case of a more general construction for 3-manifolds with boundary. A surface S in a contact manifold (M, ξ) is called *pre-Lagrangian* if its characteristic foliation is the kernel of a 1-form β on S such that $d\beta = 0$. A principal example is the boundary of a standard neighbourhood of a transverse knot (a pre-Lagrangian torus), as in the above example.

If ∂M is pre-Lagrangian then for any compatible J , $\partial M \times \mathbb{R}$ is J -flat, i.e. foliated by J -holomorphic curves. The condition of J -flatness ensures that a J -holomorphic curve in $M \times \mathbb{R}$ cannot touch the boundary $\partial M \times \mathbb{R}$ in its interior, essentially by a maximum principle; it is also a useful condition for gluing.

Moreover, a pre-Lagrangian torus boundary is standard, in the following sense: assuming that the characteristic foliation has rational slope s , the open manifold $M \setminus \partial M$ has an end $\partial M \times \mathbb{R}$ contactomorphic to a deleted neighbourhood of the z -axis in \mathbb{R}^3/\mathbb{Z} , the quotient of the cylindrically symmetric contact \mathbb{R}^3 by unit translation along the z -axis. For cylindrically symmetric solid torus neighbourhoods of the z -axis in \mathbb{R}^3/\mathbb{Z} have boundary characteristic foliations of slopes which shrink to 0 as the radius shrinks to 0; and we can map the torus ends to each other, taking the limiting slope s on $\partial M \times \{\infty\}$ to the limiting slope 0 in \mathbb{R}^3/\mathbb{Z} .

Recall that our “transverse knot contact homology” counts J -holomorphic curves in the symplectization of a closed contact 3-manifold M and keeps track of how they intersect $K \times \mathbb{R}$. But this is equivalent to considering the manifold with the knot removed — hence a manifold with a pre-Lagrangian torus end — and counting J -holomorphic curves with not only Reeb asymptotics, but certain specified asymptotics near the pre-Lagrangian boundary. Since pre-Lagrangian tori are standard, those asymptotics can be formulated in any manifold with pre-Lagrangian torus boundary; equivalently, our manifold with pre-Lagrangian end has a compactification which adds a single transverse knot. Thus we can run the whole construction for transverse knots in this more general setting.

Theorem 3.3 *Let (M, ξ) be a contact manifold with pre-Lagrangian boundary such that the characteristic foliation on ∂M has rational slope. One can define $CH^-(M, \xi)$, which is a $\mathbb{Z}_2[t]$ -module invariant of (M, ξ) . One can also define $\widehat{CH}(M, \xi)$, a \mathbb{Z}_2 -vector space invariant.*

The invariants defined here are in a sense analogous to bordered Heegaard Floer theory; the boundary at east infinity is “flat” and foliated by Reeb trajectories; and we count curves which avoid this boundary, or keep track of the number of intersections. A major goal would be to obtain some sort of gluing theorem, analogous to recent results in bordered Heegaard Floer homology theory [21]. In the contact setting, one would consider two contact 3-manifolds with pre-Lagrangian torus boundary; one can consider gluing the manifolds together, along with contact forms and almost complex structures. One would like to relate J -holomorphic curves in the symplectization of the glued manifold to curves in the two pieces, keeping track of structure on the gluing boundary, such as the intersection number counted in the differential above, and probably also the “relative height” in the symplectization (i.e. the \mathbb{R} coordinate in $M \times \mathbb{R}$) at which curves meet, as in bordered Heegaard Floer theory. These are matters for further investigation.

3.3 Convex and flat boundary

One can also consider another contact homology construction for a 3-manifold with boundary, in some sense “opposite” to the pre-Lagrangian case. We consider contact 3-manifolds with *convex* boundary; indeed, the theory of convex surfaces [8] gives a natural and generic contact boundary condition for 3-manifolds. Given the analogy with sutured manifolds, we will call (M, Γ, ξ) a *sutured contact manifold* if ξ is a contact structure on M such that ∂M is convex with dividing set Γ .

However, to define a useful contact homology for 3-manifolds with boundary, we also need to control the analytic behaviour of pseudo-holomorphic curves near the boundary of $M \times \mathbb{R}$. As in the pre-Lagrangian case, one would like to make the “east infinity” boundary $\partial M \times \mathbb{R} \subset M \times \mathbb{R}$ J -flat. This can be done; this combinatorial object can be made analytically nice.

Theorem 3.4 *Let (M, Γ, ξ) be a sutured contact 3-manifold. Let $\Sigma = \partial M$ and let Γ divide ∂M into positive and negative regions Σ_{\pm} . Then, possibly after a C^{∞} -small perturbation of Σ , there exists a contact form α on M and an almost complex structure J on $M \times \mathbb{R}$ such that:*

- (i) *The Reeb vector field R is nicely adjusted to Γ on Σ , in the following sense: R points out of M along Σ_+ ; into M along Σ_- ; and along Γ itself, so that Γ consists of Reeb trajectories.*
- (ii) *The boundary $\partial M \times \mathbb{R}$ is J -flat.*

The construction in the theorem above allows us to begin to define a “sutured contact homology”, analogous to sutured Floer homology — although work is currently in progress to show it is independent of various choices made along the way. We consider a chain complex generated by Reeb orbits away from the boundary, and consider a standard contact homology differential d counting rigid J -holomorphic curves in the symplectization.

Proposition 3.5 *One can define the “sutured contact homology” of (M, Γ, ξ) depending only on the choices of:*

- (i) *a contact form $\alpha|_{\partial M}$ with Reeb vector field nicely adjusted to Γ ;*
- (ii) *an almost complex structure $J|_{\partial M \times \mathbb{R}}$, such that $\partial M \times \mathbb{R}$ is J -flat.*

This gives a \mathbb{Z}_2 -vector space $SCH(M, \Gamma, \xi, \alpha|_{\partial M}, J|_{\partial M \times \mathbb{R}})$.

Question 3.6 *Can $SCH(M, \Gamma, \xi)$ be defined independently of α near ∂M and J near $\partial M \times \mathbb{R}$?*

This approach naturally leads to the idea of understanding contact homology in the spirit of Morse theory. We take some Morse function f and consider the subsets M_c of M defined by $f \leq c$ for values c ; we wish to understand the contact homology of (M, ξ) by understanding the sutured contact homology of the various $(M_c, \xi|_{M_c})$. We first note that a Morse function can be nicely adjusted to a contact structure, having a complete gradientlike contact vector field; this is the concept of a *convex contact structure* in [5, 8]. All regular level sets are then convex.

Corollary 3.7 *Up to the choices discussed above, for every regular value c of a Morse function f on a convex contact manifold (M, ξ) , the sutured contact homology of M_c is well-defined.*

The contact homology, as defined above, does not change in any interesting way for the corresponding M_c unless either M_c change topologically by handle additions; or the contact structure is altered, corresponding to bypass attachments [12] or “convex movies” [9].

Question 3.8 *Consider either of the following situations.*

- (i) *Suppose c is a critical value of the Morse function f , and take $c_1 < c < c_2$ close to c such that f has no critical points in $[c_1, c) \cup (c, c_2]$.*
- (ii) *Suppose f is regular on $[c_1, c_2]$ and the dividing sets $\Gamma_{\partial M_{c_1}}, \Gamma_{\partial M_{c_2}}$ differ by a bypass attachment.*

What is the relationship between the sutured contact homology of M_{c_1} and M_{c_2} ?

We have found a useful model for 1-handle additions, but things are more difficult for 2-handle additions.

One would eventually like to consider a gluing or splitting situation; splitting a contact 3-manifold M along a convex surface S , we can define the sutured contact homology of each component. These count holomorphic curves which lie on one side of $S \times \mathbb{R}$. We would like to consider curves in the symplectization of M and split along $S \times \mathbb{R}$, degenerating in some fashion so that we obtain objects associated to either side. This would amount to some “bordered contact homology” equivalent to the bordered Heegaard Floer theory of [21]. What are the correct objects to consider? And how can they be algebraized? These are important questions.

As can be seen, there are many questions which this general framework which deserve attention, and many avenues along which I would like to proceed.

References

- [1] F. Bourgeois, Y. Eliashberg, H. Hofer, K. Wysocki, and E. Zehnder, *Compactness results in symplectic field theory*, *Geom. Topol.* **7** (2003), 799–888 (electronic). MR MR2026549 (2004m:53152)
- [2] Frédéric Bourgeois, *Introduction to contact homology*, unpublished notes, <http://homepages.ulb.ac.be/~fbourgeo/pspdf/Berder.pdf>, 2003.
- [3] Yuri Chekanov, *Differential algebra of Legendrian links*, *Invent. Math.* **150** (2002), no. 3, 441–483. MR MR1946550 (2003m:53153)
- [4] Y. Eliashberg, A. Givental, and H. Hofer, *Introduction to symplectic field theory*, *Geom. Funct. Anal.* (2000), no. Special Volume, Part II, 560–673, GAFA 2000 (Tel Aviv, 1999). MR MR1826267 (2002e:53136)
- [5] Yakov Eliashberg and Mikhael Gromov, *Convex symplectic manifolds*, *Several complex variables and complex geometry*, Part 2 (Santa Cruz, CA, 1989), *Proc. Sympos. Pure Math.*, vol. 52, Amer. Math. Soc., Providence, RI, 1991, pp. 135–162. MR MR1128541 (93f:58073)
- [6] John B. Etnyre, *Lectures on open book decompositions and contact structures*, *Floer homology, gauge theory, and low-dimensional topology*, *Clay Math. Proc.*, vol. 5, Amer. Math. Soc., Providence, RI, 2006, pp. 103–141. MR MR2249250 (2007g:57042)
- [7] John B. Etnyre, Lenhard L. Ng, and Joshua M. Sabloff, *Invariants of Legendrian knots and coherent orientations*, *J. Symplectic Geom.* **1** (2002), no. 2, 321–367. MR MR1959585 (2004a:57033)
- [8] Emmanuel Giroux, *Convexité en topologie de contact*, *Comment. Math. Helv.* **66** (1991), no. 4, 637–677. MR MR1129802 (93b:57029)
- [9] ———, *Structures de contact en dimension trois et bifurcations des feuilletages de surfaces*, *Invent. Math.* **141** (2000), no. 3, 615–689. MR MR1779622 (2001i:53147)
- [10] ———, *Géométrie de contact: de la dimension trois vers les dimensions supérieures*, *Proceedings of the International Congress of Mathematicians, Vol. II (Beijing, 2002)* (Beijing), Higher Ed. Press, 2002, pp. 405–414. MR MR1957051 (2004c:53144)
- [11] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, *Invent. Math.* **82** (1985), no. 2, 307–347. MR MR809718 (87j:53053)
- [12] Ko Honda, *On the classification of tight contact structures. I*, *Geom. Topol.* **4** (2000), 309–368 (electronic). MR MR1786111 (2001i:53148)
- [13] ———, *Gluing tight contact structures*, *Duke Math. J.* **115** (2002), no. 3, 435–478. MR MR1940409 (2003i:53125)
- [14] Ko Honda, William H. Kazez, and Gordana Matić, *Convex decomposition theory*, *Int. Math. Res. Not.* (2002), no. 2, 55–88. MR MR1874319 (2002k:57064)
- [15] ———, *The contact invariant in sutured Floer homology*, <http://arxiv.org/abs/0705.2828>, 2007.
- [16] ———, *On the contact class in Heegaard Floer homology*, <http://arxiv.org/abs/math/0609734>, 2007.
- [17] ———, *Contact structures, sutured Floer homology and TQFT*, <http://arxiv.org/abs/0807.2431>, 2008.
- [18] András Juhász, *Holomorphic discs and sutured manifolds*, *Algebr. Geom. Topol.* **6** (2006), 1429–1457 (electronic). MR MR2253454 (2007g:57024)
- [19] ———, *Floer homology and surface decompositions*, *Geom. Topol.* **12** (2008), no. 1, 299–350. MR MR2390347
- [20] Robert Lipshitz, *A cylindrical reformulation of Heegaard Floer homology*, *Geom. Topol.* **10** (2006), 955–1097 (electronic). MR MR2240908 (2007h:57040)
- [21] Robert Lipshitz, Peter Ozsvath, and Dylan Thurston, *Bordered Heegaard Floer homology: Invariance and pairing*, <http://arxiv.org/abs/0810.0687>, 2008.
- [22] Daniel Mathews, *Chord diagrams, topological quantum field theory, and the sutured floer homology of solid tori*, <http://arxiv.org/abs/0903.1453>, 2009.
- [23] Lenhard L. Ng, *Computable Legendrian invariants*, *Topology* **42** (2003), no. 1, 55–82. MR MR1928645 (2003h:57038)
- [24] Peter Ozsváth and Zoltán Szabó, *Holomorphic disks and three-manifold invariants: properties and applications*, *Ann. of Math. (2)* **159** (2004), no. 3, 1159–1245. MR MR2113020 (2006b:57017)
- [25] ———, *Holomorphic disks and topological invariants for closed three-manifolds*, *Ann. of Math. (2)* **159** (2004), no. 3, 1027–1158. MR MR2113019 (2006b:57016)
- [26] ———, *Heegaard Floer homology and contact structures*, *Duke Math. J.* **129** (2005), no. 1, 39–61. MR MR2153455 (2006b:57043)