

1 Research Publications

Preprint

Catalan Numbers, Topological Quantum Field Theory, and the Sutured Floer Homology of Solid Tori

<http://arxiv.org/abs/0903.1453>

We investigate contact elements in the sutured Floer homology of solid tori, as part of the (1+1)-dimensional TQFT defined by Honda–Kazez–Matić. We find that these sutured Floer homology vector spaces form a “categorification of Pascal’s triangle”, a triangle of vector spaces, with contact elements corresponding to chord diagrams and forming distinguished subsets of order given by the Narayana numbers. We find natural “creation and annihilation operators” which allow us to define a QFT-type basis consisting of contact elements. We show that sutured Floer homology in this case reduces to the combinatorics of chord diagrams. We prove that contact elements are in bijective correspondence with comparable pairs of basis elements with respect to a certain partial order, and in a natural and explicit way. We use this to extend Honda’s notion of contact category to a 2-category. We also prove numerous results about the structure of contact elements, investigate various algebraic structures arising, and give numerous contact-geometric applications and interpretations.

From Algebra to Geometry: A Hyperbolic Odyssey — The Construction of Geometric Cone-Manifold Structures with Prescribed Holonomy.

Masters thesis, University of Melbourne, Australia.

Supervisor: Craig Hodgson

We examine the relationship between geometric cone-manifold structures on surfaces, and algebraic representations of the fundamental group into a group of isometries. A geometric cone-manifold structure on a surface, with all interior cone angles being integer multiples of 2π , determines a holonomy representation of the fundamental group. We ask, conversely, when a representation of the fundamental group is the holonomy of a geometric cone-manifold structure. We consider 2-dimensional hyperbolic geometry and expand upon the known results. We prove results for the punctured torus and higher genus surfaces. We use various ideas to allow us to deduce facts about the geometry of a representation, from algebraically derivable data. Central to these techniques are the Euler class of a representation, a geometric concept with an algebraic description, and the universal covering group $\widetilde{PSL}_2\mathbb{R}$ of the group of orientation-preserving isometries of \mathbb{H}^2 .

Mahler’s Unfinished Symphony: Études in Knots, Algebra and Geometry.

Honours thesis, University of Melbourne, Australia.

Supervisor: Craig Hodgson

This project investigates representation varieties of some knot groups, some of their geometric interpretations, and takes some pleasant detours along the way. The central investigations are of A-polynomials and parabolic representations of knots, particularly twist knots, and geometric interpretations of them. Essential to the geometric interpretation of A-polynomials is the notion of Mahler measure, which when interpreted appropriately describes the volume associated to a representation. We make various calculations involving twist knots, and more generally 2-bridge knots, and show that certain A-polynomials are irreducible.

2 Other publications

Recreational articles

In 2004 I wrote a series of recreational articles in a column entitled “mathellaneous”, for the Australian Mathematical Society Gazette.

A Beautiful Sequence

Aust. M. S. Gazette, Vol. 31 No. 1 (2004)

On an interesting sequence related to Fibonacci numbers, the game of Wythoff’s Nim, the game of Euclid, Beatty sequences, and the golden ratio.

Games with Galois

Aust. M. S. Gazette, Vol. 31 No. 2 (2004)

Reprinted (extended version) in Vinculum, a journal of the Mathematical Association of Victoria (2007)

On solving polynomial equations and playing games where winning or losing is solving or not solving.

Quadratic Geography, Algebraic Extreme Sports and Magical Farey Trees

Aust. M. S. Gazette, Vol. 31 No. 3 (2004)

A whimsical take on Conway’s theory of quadratic forms.

Knot Man

(comic; illustrator Priscilla Brown)

Paradox, the magazine of the Melbourne University Mathematics and Statistics Society.

Brimming with more as much mathematical content as a comic strip allows, Knot Man is full of audacious mathematical puns and geeky humour. The mild-mannered Theodore J. Knott, in times of mathematical emergency, with his topological utility belt and supply of high-energy genus-1 donuts, becomes Knot Man, Defender of the Mathematical Universe.

Mathematical Olympiad

In progress

Mathematical Olympiad Lectures

(Joint with A. Di Pasquale and N. Do)

This book is based on lectures given to Australian high school students involved in the mathematical olympiad training programmes in the country. It will be published by Australian Mathematics Trust Publishing.