Math 53 Su10, Problem Set 3
July 2, 2010

Do not hand in solutions, but be sure to understand how to do all problems. Some of these will be put on Friday’s quiz.

1. Consider the initial value problem given by

\[ y' = \begin{cases} 
1 & \text{if } y > 0 \\
0 & \text{if } y = 0 \\
1/y & \text{if } y < 0 
\end{cases} \]

\[ y(0) = 0 \]

(a) What does the existence theorem say about solutions to this initial value problem?
(b) Can you construct a solution to the initial value problem? If yes, do so. If not, explain why not.
(c) Explain briefly why your answers in 1a and 1b don’t contradict anything.

2. Consider the differential equation:

\[ y' = y^2 \sin y. \]

(a) Describe the behavior as \( t \to +\infty \) of the solution with initial value \( y(0) = -1 \).
(b) For each possible choice of \( y_0 \), determine the interval of existence (interval of validity/interval of definition) for a solution with initial value \( y(0) = y_0 \).
(c) Suppose this equation comes to us from a model for a physical system. We have experimental data that tells us that \( y(0) \) is somewhere in the interval \((-1/100, 1/100)\). Is it possible that \( y(100) = 1 \)? How about \( y(100) = -2 \)? (Explain your answer.)

3. (Problem 39 on page 48 of the text.)
In this problem, we will develop the method of variation of parameters.

(a) i. Consider the equation:

\[ y' = p(t)y \]

Show the general solution has the form

\[ y(t) = A e^{\int p(t)dt}. \]

ii. Now consider the equation:

\[ u' = p(t)u + q(t). \]

Look for a solution of the form

\[ u(t) = A(t) e^{\int p(t)dt}. \]

By substituting this into the differential equation, find an equation that \( A(t) \) must satisfy. Use this equation to find \( u(t) \).

(b) Use the method of variation of parameters to solve the equation:

\[ y' - 2y = e^{2t} t^3 \]
4. Solve the following system of differential equations:

\[
\begin{align*}
y_1' &= y_1 - 2y_2 \\
y_2' &= y_1 + 4y_2
\end{align*}
\]

Suggested problems:
Page 79 : 22, 23
Page 92 : 7, 15, 20 (you may also find 21, 22 and 23-25 interesting).
Page 146 : 21
Page 162 : 2, 3, 26.