Due Thursday, December 3rd

Submit your solutions in discussion section. Be sure to show all your work!

- Section 7.1 : 8a,b,d; 10; 14
- Section 7.2 : 12, 22
- Section 7.3 : 12, 16, 40, 46, 50

This problem will illustrate another model using differential equations. We have an artificial fish pond whose basin is a hemisphere, 10 meters in radius. We want to study the water depth as a function of time. Denote the depth of the water by \( y(t) \).

1. Find the volume of the water in the pond as a function of the depth \( y \). Find the surface area of the pond as a function of \( y \). (Assume \( y \leq 10 \text{m} \) so we don’t have to worry about overflowing.)
2. Assume the evaporation rate is proportional to the surface area of the pond, at a rate of \( r = 0.0085 \text{m/day} \) (i.e. we lose 0.0085 cubic meters of water to evaporation per day, per square meter of surface area). Write down a differential equation to model the depth of the water \( y(t) \) as a function of time. (Hint: start with the volume and the area, and use the previous question.)
3. You want to install a water source that feeds water in at a constant rate \( q \). Write down the differential equation for \( y(t) \) in this new situation. Find the value of \( q \) that enables us to keep a constant water level of \( y = 9 \text{m} \).
   What happens if we start with a lower water level? How about a higher water level?

Bonus: (Hand in the solution to the bonus separately from the rest of the homework, directly to Sam Lisi, anytime before 5 PM on Friday, December 4th.)

(a) Refer back to the differential equation you found in part (3) (with evaporation, but no replacement of the water). After simplification, this equation takes a surprisingly simple form. Explain why it is so simple.

(b) Suppose now that instead of a hemispherical basin, the pond actually has an irregularly shaped basin. Find and justify a differential equation for \( y \) as a function of time, with evaporation (but without replacing the water). (Hint: use the type of reasoning you used in (a). It may be convenient to introduce the function \( V(y) \) that gives the volume of water corresponding to depth \( y \).)