SIMPLE SINGULARITIES AND INTEGRABLE HIERARCHIES

TODOR E. MILANOV

This is a joint work with A. Givental. I will not be assuming that people are familiar with integrable systems or singularity theory.

Let $f : \mathbb{C}^3 \to \mathbb{C}$ be a holomorphic function with an isolated critical point at 0 of A, D, or E type. Using the theory of vanishing cycles and period mappings, we will construct an integrable hierarchy in terms of vertex operators and Hirota quadratic equations (shortly HQE). Such descriptions link the theory of integrable systems to infinite dimensional geometry. More precisely, via the Boson–Fermion correspondence, the HQE turn into an infinite system of quadratic equations (which resemble the classical Plücker relations), and thus the solutions of the HQE form an infinite dimensional submanifold of the projective space $\mathbb{P}(\wedge^\bullet(\mathbb{C}^\infty))$. In our case, these submanifolds can be described explicitly: they coincide with certain orbits of certain representations of the corresponding affine Kac–Moody Lie groups of A, D, or E type.

In this talk, I am planning to describe a deformation of the HQE, governed by the Frobenius structure on the space of miniversal deformations of the function f. After deforming f, its critical point decomposes into N Morse type critical points. The deformed HQE, in some sense, are equivalent to N copies of the HQE corresponding to the A_1 critical point (i.e. Morse type critical point). The integrable system corresponding to the A_1 critical point is the famous KdV hierarchy. Now I can summarize the main results as follows: with each critical point of A, D, or E type we can associate an integrable hierarchy, which admits a solution in terms of the deformation theory of f and solutions to the KdV hierarchy.

In a subsequent talk, I will show how the methods explained above can be used to prove both the equivariant and the non-equivariant Toda conjectures in the Gromov–Witten theory of $\mathbb{C}P^1$. It will be interesting to see if the above methods can be generalized to other singularities as well. If such generalization exists, then we will see some knew integrable systems, representations of some unknown Lie groups, and most importantly we will be able to construct integrable systems which govern the Gromov–Witten invariants of symplectic manifolds.

E-mail: milanov@math.stanford.edu.