Let $F$ be a totally real field. A Hilbert cuspidal eigenform $f$ over $F$ of weights $\geq 2$ gives rise to an $\ell$-adic representation $\text{Gal}(\overline{F}/F) \to \text{GL}_2(\overline{\mathbb{Q}}_\ell)$ for each prime $\ell$. The local behavior of these representations at each prime $p$ of $F$ is captured in a family of Weil-Deligne representations $\rho_{f,p}: W_{F_p} \to \text{GL}_2(\mathbb{C})$, unramified for $p$ not dividing the level of $f$ and satisfying the usual trace and determinant conditions.

Let another family $\{\rho_p\}$ be given which is unramified almost everywhere, and let a vector of weights $k$ be given satisfying a certain compatibility condition with respect to $\{\rho_p\}$. We show that $f$ can almost always be chosen so that $\rho_{f,p}$ matches $\rho_p$ on inertia. (“Almost always” means there are finitely many exceptional cases up to twisting.)