

MATH 155: PROBLEM SET 7

DUE DECEMBER 3

1. Find the four dimensional escalators of
 - (a) $x^2 + 2y^2 + 4z^2$
 - (b) $x^2 + 2y^2 + 2yz + 4z^2$,and
 - (c) $x^2 + y^2 + 3z^2$.
2. Let d be a positive square-free number. If $x^2 - dy^2 = n$ has an integer solution with $xy \neq 0$, show that it has infinitely many.
3. Let $a + b\sqrt{p}$ be the fundamental solution to Pell's equation $x^2 - py^2 = 1$ where $p \equiv 1 \pmod{4}$ is prime. The following exercise shows that $x^2 - py^2 = -1$ has a solution.
 - (a) Show that a is odd.
 - (b) Show that $a \pm 1 = 2u^2$, $a \mp 1 = 2pv^2$, $2uv = b$.
 - (c) $u^2 - pv^2 = \pm 1$.
 - (d) The $-$ sign holds in (c).