MATH 155: PROBLEM SET 2

Due October 15

1. Verify the following assertions made in class: The smallest number primitively represented by a reduced form $f(x, y) = ax^2 + bx + c$ is a, and the next number primitively represented is c. Complete all details of the proof that two reduced forms are inequivalent.

2. Show that the numbers n of the form $x^2 + 3y^2$ are precisely those that can be written as nm^2 where the prime factors of n are all from the set $\{3, p \equiv 1 \pmod 3\}$.

3. Prove that the product of two numbers of the form $x^2 + 5y^2$ is also a number of the same form. Prove that the product of two numbers of the form $2x^2 + 2xy + 3y^2$ is a number of the form $x^2 + 5y^2$. Prove that the product of a number of the form $x^2 + 5y^2$ and a number of the form $2x^2 + 2xy + 3y^2$ is of the form $2x^2 + 2xy + 3y^2$.

4. Characterize (similarly to problem 2) all numbers of the form $x^2 + 5y^2$.

5. Describe the two dimensional lattice associated to the quadratic form $7x^2 + 23xy + 20y^2$. Give a basis, and the lengths of the vectors in the basis and the angle between them. What is the length of the shortest vector in this lattice? What is the bound furnished for this length by Minkowski's theorem?