MATH 155: PROBLEM SET 3

Due February 4

1. In class we sketched a proof that

$$\frac{\phi(q)}{q} \geq \frac{e^{-\gamma} + o(1)}{\log\log q},$$

and that there are arbitrarily large values for which (asymptotic) equality here is attained. Supply a complete proof.

2. Prove that the 3-divisor function $d_3(n) \leq C(\epsilon)n^{\epsilon}$ for any $\epsilon > 0$ and some positive constant $C(\epsilon)$. Find an explicit constant C such that the inequality $d_3(n) \leq Cn^{1/4}$ holds for all integers n.

3. In class we showed starting from the polynomial $P_N(x) = (x(1-x))^N$ and considering $\int_0^1 P_N(x)dx$ that $\psi(2N+1) \geq 4^N$. Follow through that argument with $P_N(x) = (x^2(1-x)^2(2x-1))^{2N}$ and obtain a lower bound for $a = \liminf_{x\to\infty} \psi(x)/x$.