MATH 155: MIDTERM EXAMINATION

February 2, 2010

This is a closed book, closed notes test. You are free to use results proved in class, but you must state clearly the result that you are using. Complete proofs are expected for all questions. There will be partial credit, so please include approaches to the problem, and/or heuristic reasons for what the result might be or why it might be true. Don't get too bogged down by error terms; if you can prove an asymptotic without a very good error term, that still counts for a lot. All the best!

1. Find an asymptotic for the number of positive integers below x that have a prime factor larger than $\sqrt{x}$.

2. Let $\sigma(n)$ denote the sum of the divisors of n; thus $\sigma(6) = 1 + 2 + 3 + 6 = 12$. Find an asymptotic formula for $\sum_{n \leq x} \sigma(n)$.

3. Let $f(n) = \sum_{p|n} (\log p)^2$. Find an asymptotic formula for

$$\sum_{n \leq x} f(n).$$

4. A number is called cube-free if it is not divisible by the cube of any prime. Show that $\sum_{d^3|n} \mu(d)$ equals 1 if n is cube-free and 0 otherwise. How many cube-free numbers are there up to x?