## MATH 155: PROBLEM SET 6

Due May 17

1. Using the method of Perron's formula and contour integration derive an asymptotic formula for the number of $k$-free integers up to $x$. Note: a number is $k$-free if it is not divisible by the $k$-th power of any prime. Here $k \geq 2$ is an integer.
2. Prove that there is a constant $c>0$ such that

$$
\sum_{n \leq x}\left(\frac{n}{\phi(n)}\right)^{2012}=c x+O\left(x^{1-\delta}\right)
$$

for some $\delta>0$.
3. Consider the series

$$
F(s)=\sum_{n=1}^{\infty} \frac{1}{\phi(n)^{s}}
$$

Where does this converge absolutely? Show that $F(s)$ extends meromorphically to the region $\operatorname{Re}(s)>0$, and is analytic except a simple pole at $s=1$. Make a guess about how many integers $n$ are there with $\phi(n) \leq x$. What issues would you face if you want to make a rigorous proof of your guess?
4. In class we discussed the problem of determining an asymptotic for $\sum_{n \leq x} a(n)$ where $a(n)$ is the number of abelian groups of order $n$. Give a complete proof of such an asymptotic formula.

