MATH 155: PROBLEM SET 2

Due April 19

1. Let d_x denote the least common multiple of the integers not exceeding x. Show that

$$\binom{2n}{n} = \prod_{k=1}^{\infty} d_{2n/k}^{(-1)^{k-1}}.$$

What can you say about the size of d_x?

2. Prove directly the identity from class

$$\mu(n) \log n = -\sum_{ab=n} \mu(a)\Lambda(b).$$

3. Let f and g be two multiplicative functions. Show that $f*g$ is also multiplicative. A function is called completely multiplicative if it satisfies $f(mn) = f(m)f(n)$ for all m and n. Is the convolution of two completely multiplicative functions necessarily completely multiplicative?

4. Prove that

$$2^{\omega(n)} = \sum_{d^2 m = n} \mu(d) d(m).$$

(Hint: One way to check such identities is to prove them for prime powers, and then use multiplicativity.) Prove that

$$\sum_{n \le x} 2^{\omega(n)} = \frac{6}{\pi^2} x \log x + cx + O(\sqrt{x} \log x),$$

for some constant c. You may use without proof that $\zeta(2) = \pi^2/6$.