MATH 152: PROBLEM SET 1

Due Wednesday, September 29

1. Show that $n^4 + n^2 + 1$ is not prime for every $n > 1$.

2. Irrational numbers. The following are in ascending order of difficulty: although the last part contains all others you may want to do the parts in order to get an idea of how to prove that.

(i) Show that $\sqrt{p}$ is irrational for any prime p.

(ii) Show that $\sqrt{n}$ is irrational unless n is the square of an integer.

(iii) Suppose α is a solution to the polynomial equation $x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n = 0$ where $a_1, \ldots, a_n$ are integers. Show that either α is an integer or α is irrational.

3. Show that for every natural number $n \geq 1$,

$$\frac{(30n)!n!}{(15n)!(10n)!(6n)!}$$

is an integer.

4. Prove that $a|bc$ if and only if $\frac{a}{(a,b)}|c$.

5. Let $n \geq 2$ be a natural number, and consider the n integers $j(n!) + 1$ where $1 \leq j \leq n$. Show that any two of these numbers are relatively prime (that is, have g.c.d. equal to 1). Use this observation to give another proof that there are infinitely many primes.

6. If 2010! were written out in the ordinary decimal notation, how many zeros in a row would there be at the right end?