MATH 152: MIDTERM

In class: Closed Book and Closed Notes

NOTE: Proofs/explanations are needed for all problems. All the best!

1. Consider the group of reduced residue classes $\pmod{1001}$. (Note $1001 = 7 \times 11 \times 13$). What is the largest possible order of an element of this group? You must also prove that there are elements of this order.

2. Let $\ell \geq 2$ be a natural number and let p be a prime with $p \equiv 1 \pmod{\ell}$. Consider the congruence $x^\ell \equiv a \pmod{p}$ for $(a, p) = 1$. Prove that there are $(p-1)/\ell$ reduced residue classes a for which this congruence has ℓ solutions, and for the remaining reduced residue classes the congruence has no solutions.

3. Is it true that there is a rational number x with $|x|_2 \geq 1024$, $|x-1|_3 \leq 1/27$ and $|x-2|_5 = 25$? Here $|x|_p$ denotes the p-adic absolute value of x. You must explain your answer.

4. (a). Let n be an odd natural number with $n \equiv 5 \pmod{13}$. Prove that the congruence $x^2 \equiv 13 \pmod{n}$ has no solutions.

(b). Suppose n is odd and $n \equiv 1 \pmod{13}$. Is it necessarily true that the congruence $x^2 \equiv 13 \pmod{n}$ has a solution?