

## MASTERCLASS PROBLEM SUGGESTIONS

1. (1997 : A4) Let  $G$  be a group with identity  $e$  and let  $\phi : G \rightarrow G$  be a function such that

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

whenever  $g_1g_2g_3 = e = h_1h_2h_3$ . Prove that there exists an element  $a \in G$  such that  $\psi(x) = a\phi(x)$  is a homomorphism (that is,  $\psi(xy) = \psi(x)\psi(y)$  for all  $x, y \in G$ ).

2. (IMO 1991) Given a real number  $a > 1$  construct a bounded infinite sequence of real numbers  $x_0, x_1, x_2, \dots$  such that

$$|x_i - x_j| \cdot |i - j|^a \geq 1$$

for all distinct non-negative integers  $i, j$ .