

MASTERCLASS WEEK 6: PROBLEMS

1. (Olena Bormashenko; 2005 A3) Let $p(z)$ be a polynomial of degree n all of whose zeros have absolute value 1 in the complex plane. Put $g(z) = p(z)/z^{n/2}$. Show that all the zeros of $g'(z)$ have absolute value 1.
2. (Thanhthanh Tran; 1999 A5) Prove that there is a constant C such that if $p(x)$ is a polynomial of degree 1999 then

$$|p(0)| \leq C \int_{-1}^1 |p(x)| dx.$$

3. (Natth Bejraburnin; IMO 1983) Do there exist 1983 positive integers less than 100000 no three of which lie in an arithmetic progression?
4. (Daniel Le) Suppose p and q are two nonconstant polynomials with complex coefficients such that $p(x)$ and $q(x)$ have the same roots (precisely, the roots are the same but their multiplicities need not be), and moreover $p(x) - 1$ and $q(x) - 1$ have the same roots (again not necessarily with the same multiplicities). Prove that $p(x) = q(x)$.