

MASTERCLASS WEEK 3: PROBLEMS

1. (Elina Robeva: IMO 1988) Show that the set of real numbers x that satisfy the inequality

$$\sum_{k=1}^{70} \frac{k}{x-k} \geq \frac{5}{4}$$

is a union of disjoint intervals, the sum of whose lengths is 1988.

2. (Deyan Simeonov: IMO 1991) Let $S = \{1, 2, \dots, 280\}$. Find the smallest integer n such that each n -element subset of S contains five numbers that are pairwise coprime.

3. (Nathan Pflueger : 1989 A6) Let $\alpha = 1 + a_1x + a_2x^2 + \dots$ be a formal power series in the field of two elements with $a_n = 1$ if every block of zeros in the binary expansion of n has an even number of zeros in the block, and $a_n = 0$ otherwise. Prove that $\alpha^3 + x\alpha + 1 = 0$ (as a power series in \mathbb{F}_2).

4. (Bob Hough) Given z_1, \dots, z_n complex numbers show that there is a subset I of $\{1, 2, \dots, n\}$ such that

$$\left| \sum_{i \in I} z_i \right| \geq \frac{1}{\pi} \sum_{i=1}^n |z_i|.$$