MASTERCLASS WEEK 3: PROBLEMS

1. (Boris Hanin; USAMTS, 2002-2003) A fudgeflake is a planar fractal figure with 120 degree rotational symmetry such that three identical fudgeflakes in the same orientation fit together without gaps to form a larger fudgeflake with its orientation 30 degrees clockwise of the smaller fudgeflakes' orientation (Boris will draw a figure!). If the distance between the centers of the original three fudgeflakes is 1 what is the area of one of those three fudgeflakes?

2. (Sound) Among the numbers 2^n $(1 \leq n \leq 10^6)$ how many begin with the leading decimal digit 1? Among the numbers 2^n, which leading digit appears more frequently 7 or 8?

3. (Yong Suk Moon; A5 1989) Let m be a positive integer and let $\mathcal{G}$ be a regular $(2m+1)$-gon inscribed in the unit circle. Show that there is a positive constant A, independent of m, with the following property. For any point p inside $\mathcal{G}$ there are two distinct vertices v_1 and v_2 of $\mathcal{G}$ such that

$$\Big||p - v_1| - |p - v_2|\Big| < \frac{1}{m} - \frac{A}{m^3}.$$

Here $|s - t|$ denotes the distance between the points s and t.

4. (Nathan Pflueger; from Leo Goldmakher) A horse breeder has 25 horses. He has a track on which he can race five horses at once. A race determines the ranking of those five horses, but does not give any further information (he cannot time the horses, for instance). Assume the horses can be ranked first through last without ties and that a faster horse always defeats a slower horse. How many races are necessary to determine the top three horses? It is not necessary to determine the ranking of the top three, only to determine which three horses are the fastest.