1 Multivariable functions and parametric curves

Example 1.1 Consider in $\mathbb{R}^2$ the sets

\[ A = \{(7 - t^3)(2, 1) \mid t \in \mathbb{R}\} \]
\[ B = \{(-2, 2) + t^5(1, 1) \mid t \in \mathbb{R}\}. \]

What geometric object do these sets represent? Find their intersection points in $\mathbb{R}^2$, if there are any.

Example 1.2 Consider in $\mathbb{R}^3$ the sets

\[ A = \{t(1, 2, 3) \mid t \in \mathbb{R}\} \]
\[ B = \{(1, 0, 0) + t(1, 1, 1) \mid t \in \mathbb{R}\}. \]

Find the intersection points of $A$ and $B$, if there are any.

Example 1.3 Let $f(x,y) = x^2 + y^2$, and consider the graph

\[ G = \{(x,y,z) \in \mathbb{R}^3 \mid z = f(x,y)\}. \]

1. Consider the curve $\gamma$ in $\mathbb{R}^3$ given by $\gamma(t) = (t, 0, t^k)$. For what value of $k$ does $\gamma$ lie on $G$?

2. What is the velocity of $\gamma$ when $t = 1$? What is the acceleration of $\gamma$ when $t = 1$?

3. Find an example of another curve $\eta$ in $\mathbb{R}^3$ which lies on $G$, passes through the point $(1,0,1)$, and with velocity at that point linearly independent from the velocity of $\gamma$ at that point.

Compute the tangent plane of $G$ at the point $(1,0,1) \in G$. Hint: the linearly independent velocities from the previous parts will span the plane.
2 Limits and continuity

Example 2.1 Let $f(x, y) = x^2 + y^2$. What is the limit of $f$ as $(x, y)$ approaches $(2, 2)$, i.e.

$$\lim_{(x,y) \to (2,2)} f(x, y)$$

Exercise 2.2 Compute

$$\lim_{(x,y,z) \to (2,1,-1)} 3x^2 z + y x \cos (\pi x - \pi z).$$

Exercise 2.3 Compute

$$\lim_{(x,y) \to (5,1)} \frac{xy}{x + y}.$$

Exercise 2.4 Compute

$$\lim_{(x,y) \to (0,0)} \frac{x^2 y^2}{x^4 + 3y^4}.$$

Exercise 2.5 Compute

$$\lim_{(x,y) \to (0,0)} \frac{x^3 y}{x^6 + y^2}.$$