1 More on determinants

Proposition 1.1 Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with matrix $A$, and let $R$ be a region in $\mathbb{R}^2$. Then the area of $T(R)$ is $|\det(A)|$ times the area of $R$.

Exercise 1.2 Find the area of the parallelogram formed by $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

Exercise 1.3 Let $R$ be the region in $\mathbb{R}^2$ enclosed by the curve
\[
\frac{(2x + y)^2}{4} + \frac{(x + 3y)^2}{9} = 1.
\]

2 Systems of coordinates

Recall that if $\mathcal{B} = \{v_1, v_2, \ldots, v_k\}$ is a basis for a subspace $V$ of $\mathbb{R}^n$, and we have
\[
v = c_1 v_1 + c_2 v_2 + \cdots + c_k v_k,
\]
then we call the coefficients $c_1, c_2, \ldots, c_k$ the coordinates of $v$ with respect to the basis $\mathcal{B}$, and we write
\[
[v]_{\mathcal{B}} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{pmatrix}.
\]

Example 2.1 Let
\[
v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.
\]

Then $\mathcal{B} = \{v_1, v_2\}$ is a basis for $\mathbb{R}^2$. 

• Find the \( B \) coordinates of

\[
x = \begin{pmatrix} 4 \\ 5 \end{pmatrix}.
\]

• Given that

\[
[y]_B = \begin{pmatrix} -2 \\ 3 \end{pmatrix}
\]

, find \( y \) in standard coordinates.

**Exercise 2.2** Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be the transformation with matrix

\[
A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}
\]

with respect to the standard basis, and let

\[
\mathcal{B} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.
\]

1. Find the matrix for \( T \) with respect to \( \mathcal{B} \).

2. Find \( A^{10} \).

**Exercise 2.3** Let \( V \) be the plane \( x_1 + 2x_2 + 5x_3 = 0 \) in \( \mathbb{R}^3 \). Find the matrix for the orthogonal reflection about \( V \).