Quiz 2 Solutions
Monday, January 28

1. State the definition of the derivative of \( f(x) \) at the point \( x = a \). Specifically,

Solution:

\[ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, \]

or

\[ f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}. \]

\[ \square \]

2. Which of the following is not an expression for the slope of the tangent line to the curve \( y = f(x) \) at the point \( P = (a, f(a)), (a \neq 0) \)?

Solution: (b)

\[ \lim_{x \to a} \left( \frac{f(x) - f(a)}{x} \right) \]

\[ \square \]

3. Find the equation of the tangent line to the curve \( y = x^2 + 1 \) at the point \( (1, 2) \). Feel free to use the fact that \( \lim_{x \to 1} \left( \frac{(x^2 + 1) - 2}{x - 1} \right) = 2. \)

Solution: The slope of the tangent line to the curve \( y = x^2 + 1 = f(x) \) at the point \( (1, 2) \) is given by:

\[ \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \left( \frac{(x^2 + 1) - 2}{x - 1} \right) = 2 \]

as is given.
Thus the equation is \( y - 2 = 2(x - 1) \).

\[ \square \]

4. Suppose I need to know if there a root of \( f(x) = \ln(x + 1) + \sqrt{x} - 1 \) between 0 and 2. What theorem can I use to find out if such a root exists? What are the two hypotheses of this theorem that I must check before I can apply it?
Solution: I can use the Intermediate Value Theorem to solve this problem. The two hypotheses for this theorem are:

(a) \( f(x) \) must be a continuous function on the interval \([0,2]\).

(b) The number 0 must be between \( f(0) \) and \( f(2) \).

5. Let \( f(x) = \begin{cases} 0 & : x < -2 \\ 1 & : -2 \leq x \leq 0 \\ \frac{1}{x-3} & : x > 0 \end{cases} \)

\( f(x) \) is continuous on which of the following intervals?

Solution: (b) \([-2,0]\).