Quiz 1 Solutions  
Wednesday, January 23

1. State the definition of the limit of $f(x)$, as $x$ goes to $a$. Specifically,
\[
\lim_{x \to a} f(x) = L
\]
means

*Solution:* we can make the values of $f(x)$ arbitrarily close to $L$ by taking $x$ to be sufficiently close to $a$, $x \neq a$.

2. Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$ (in other words, both limits exist and are equal to $L$ and $M$ respectively). Which of the following might *not* be true?

*Solution:*
\[
(d) \lim_{x \to a} \left( \frac{f(x)}{g(x)} \right) = \frac{L}{M}
\]
because $M$ could be 0, in which case, the division limit law does not apply.

3. Consider the limit $\lim_{x \to 0^-} |x|$. Is this a left- or right-hand limit? As $x$ approaches 0, is $x$ positive or negative? What is the value of this limit?

*Solution:* $\lim_{x \to 0^-} |x|$ is a left-hand limit. As $x$ approaches 0 in this limit, $x$ is negative. Further, $\lim_{x \to 0^-} |x| = 0$.

4. Give an example of a function $y = f(x)$ that is defined at $x = 0$, $\lim_{x \to 0} f(x)$ is a finite limit, but $\lim_{x \to 0} f(x) \neq f(0)$. Is this function continuous at $x = 0$?

*Solution:* The simplest example is:
\[
f(x) = \begin{cases} 
0 & : x \neq 0 \\
1 & : x = 0.
\end{cases}
\]
Since $\lim_{x \to 0} f(x) \neq f(0)$, the function is not continuous at $x = 0$.

5. Which of the following functions is *not* continuous at $x = 0$?

*Solution:*
\[
(c) f(x) = \begin{cases} 
1 & : x < 0 \\
x & : x \geq 0.
\end{cases}
\]
This function is not continuous at $x = 0$ because $\lim_{x \to 0} f(x)$ does not exist much less equal $f(0) = 0$. 

\[\square\]