Lecture Outline (Optimization II)
Friday, February 29

Announcements

1. Grades for midterm 2 will be posted on Coursework by Sunday evening.
2. Quiz 7 on Monday.

Optimization Problems - On Open Intervals

It often happens that you are asked to optimize a function on an open interval (or the whole real line) instead of on a closed interval. How does one go about solving such a problem? Happily, much of the technique for optimizing functions on closed intervals will carry over to optimizing functions on open intervals.

The main difference in technique is that in finding absolute maxima or minima we will need to use a variant of the first derivative test instead of the techniques we developed on Monday to find the absolute maxima and minima for continuous functions on closed intervals. This is our variation of the first derivative test:

1st Derivative Test for Absolute Minima and Maxima Suppose $f(x)$ is a continuous function defined on $(a, b)$ (where $a = -\infty$ and $b = \infty$ are possible). Let $x = c$ be a critical point of $f(x)$. Then:

1. If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f$ has an absolute minimum (in its domain) at $c$.
2. If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f$ has an absolute maximum (in its domain) at $c$.

Notice that this is very similar to the 1st derivative test for local min/max. The difference is that in this test we require information about the sign of the derivative for all $x < c$ and all $x > c$; in the local test, we needed to only know this for values of $x$ near $c$.

Note:

1. To use the 1st derivative test for absolute min/max directly, you should be in the situation where you have exactly one critical point in your domain. If you have more than one critical point in your domain, you probably have to do more work.
2. If you’re in the situation where you have exactly one critical point (say, $x = c$) in your domain, then to test the sign of the derivative for values $x < c$ it is enough to compute the sign of the derivative for one value of $x < c$. Similarly for checking the sign when $x > c$.

The technique then for solving optimization problems for continuous functions on an open interval is:

1. Read and re-read the problem until you understand it. In particular, make sure you know the quantity you’re being asked to maximize or minimize.

2. Draw and label a picture which gives the relevant information.

3. Write equations that describe
   
   (a) the quantity that you’re attempting to maximize/minimize in terms of the other variables which appear in your drawing, and
   
   (b) the constraints that your variables must obey.

4. Solve Equation (b) for one of the variables, and plug this result back into Equation (a).

5. Determine the domain of your function. This is very important!

6. Use the first derivative test for absolute minima and maxima to finish the problem.

Examples

1. Find two numbers whose sum is 23 and whose product is maximum.

2. A cylindrical can is to be made to hold $1000cm^3$ of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.