Lecture Outline (Maximum and Minimum)
Monday, February 25

Example for finding local maxima and local minima:

Suppose \( y = x^4 - 25x^2 \). Find the local minima and maxima of \( f(x) \) on \([-2, 10]\).

Theory: Absolute Maxima and Minima

How can we find absolute maxima and minima? A natural answer is to find all the local maxima and minima. Then the largest maximum is the absolute maximum and the smallest local minimum is the absolute minimum. This is almost true.

For one, the function might not even have a local maximum or a local minimum (think \( y = x^3 \)). Another example is \( y = \tan(x) \) We can see that the function does not attain either a maximum value nor a minimum value on its domain.

When can we find the absolute maximum and minimum?

**Extreme Value Theorem** Suppose \( f(x) \) is a continuous function on a closed interval \([a, b]\). The function \( f(x) \) has an absolute maximum \( c \) and an absolute minimum \( d \) for numbers \( c \) and \( d \) in \([a, b]\).

Notice that this is true when two things hold: \( f(x) \) must be continuous and it must be defined on a closed interval.

If either of these things are not true, it is possible for the function not to have an absolute maximum or an absolute minimum.

We can still answer this question but we must check that the function doesn’t get bigger/smaller at points that are not local maxima or minima.

Finding Absolute Maxima and Minima for Continuous Functions on Closed Intervals

We can use the Extreme Value Theorem to find absolute maxima and minima. Here’s a 4 step technique:

Finding Absolute Maxima and Minima
1. Verify that \( f(x) \) is continuous on a closed interval of interest

2. Find the critical points of \( f(x) \) that lie in the closed interval of interest

3. Evaluate \( f(x) \) at critical points and endpoints in a chart.

4. Pick out the absolute maximum by finding which critical point/endpoint gives you the largest function value/smallest function value.

To use this process it is critical that you verify the first step. If you don’t first check that the function is continuous on a closed interval, this method can fail! Let’s try using it.

Example: Find the absolute maxima and minima of the function \( f(x) = x\sqrt{x} - x^2 \) on the interval [0,1].