Announcements

1. Quiz 6 on Monday, 2/25/08.
2. Homework 6 due on Wednesday, 2/27/08.
3. Midterm 2 is on Thursday, 2/28/08, 7:00-9:00 PM in Herrin T175. I must know by Sunday evening you cannot take the exam at the scheduled time.
4. A review session will be held Wednesday, 2/27/08, at 7:00 in 380W.
5. Ziyu has to move his office hours next week from 7:00-10:00 on Tuesday to 7:00-10:00 PM on Monday. Plan accordingly.

Recap

Recall from Wednesday the following facts:

1. If \( f''(x) > 0 \) on an interval, then \( f(x) \) is concave up on that interval.
2. If \( f''(x) < 0 \) on an interval, then \( f(x) \) is concave down on that interval.
3. \( f(x) \) is increasing on an interval if and only if \( f'(x) > 0 \).
4. \( f(x) \) is decreasing on an interval if and only if \( f'(x) < 0 \).

**Definition:** A function has an inflection point at \( x = a \) if the concavity \( f(x) \) changes at \( a \).

Examples

1. Sketch the graph of a function that satisfies the following conditions: \( f'(1) = f'(-1) = 0 \), \( f'(x) < 0 \) if \( |x| < 1 \), \( f'(x) > 0 \) if \( 1 < |x| < 2 \), \( f'(x) = -1 \) if \( |x| > 2 \), and \( f''(x) < 0 \) if \( -2 < x < 0 \).
More Terminology

Definition: A critical point of \( f(x) \) is a point where \( f'(x) = 0 \) or \( f'(x) \) is undefined.

Definition: A function \( f(x) \) has an absolute maximum at \( x = c \) if \( f(x) \) is the largest value of \( f(x) \). The \( f(c) \) is the maximum value of \( f(x) \). Similarly, \( f(x) \) has an absolute minimum at \( x = c \) if \( f(c) \) is the smallest value of \( f(x) \). Then \( f(c) \) is the minimum value of \( f(x) \).

The function \( f(x) \) has a local maximum at \( x = c \) if \( f(c) \geq f(x) \) for all values of \( x \) near \( c \) and in the domain of \( f(x) \). Similarly, \( f(x) \) has a local minimum at \( x = c \) if \( f(c) \leq f(x) \) for all values of \( x \) near \( c \) and in the domain of \( f(x) \).

Note: my definition is slightly different from the textbooks - we’ll be using the above definition.

Finding Local Maxima and Minima

There are three possible places for local maxima and minima to occur: (Draw pictures for each)

1. At a value \( x = c \) where \( f'(x) = 0 \),
2. At a value \( x = c \) where \( f \) is not differentiable, or
3. At a value \( x = c \) that is an endpoint of the domain of \( f \).

Notice that these are critical points of the function. Note that a critical point is not always a local max or min.

Once we have a point \( x = c \) where local maxima and minima might occur, how do we determine whether the function has a local maxima, local minima, or neither at this point? We have two tests that help us out.

First Derivative Test: Suppose \( y = f(x) \) has a critical point at \( x = c \) and that \( f \) is continuous at \( x = c \).

1. If \( f'(x) > 0 \) for values of \( x \) less than \( c \) and \( f'(x) < 0 \) for values of \( x \) greater than \( c \), then \( x = c \) is a local maximum.
2. If \( f'(x) < 0 \) for values of \( x \) less than \( c \) and \( f'(x) > 0 \) for values of \( x \) greater than \( c \), then \( x = c \) is a local minimum.
3. Otherwise, \( x = c \) is neither a local maximum nor a local minimum.
Second Derivative Test: Suppose $y = f(x)$ has a critical point at $x = c$ and that $f''(c)$ is defined.

1. If $f''(c) < 0$, then $f$ has a local maximum at $x = c$.
2. If $f''(c) > 0$, then $f$ has a local minimum at $x = c$.
3. If $f''(c) = 0$, we can’t say anything with this test.

Examples

1. Find the local maxima and minima of $f(x) = x^3 + 3/2x^2 - 18x + 1$. 