Lecture Outline (What $f'$ says about $f$.)
Wednesday, February 20

Announcements

1. Homework 6 due on Wednesday, 2/27/08.

2. Midterm 2 is on Thursday, 2/28/08, 7:00-9:00 PM. A review session will be held Wednesday, 2/27/08, at 7:00 in 380W.

3. Ziyu has to move his office hours next week from 7:00-10:00 on Tuesday to 7:00-10:00 PM on Monday. Plan accordingly.

Recap

Recall from Friday the following facts:

\[ f(x) \text{ is increasing on an interval if and only if } f'(x) > 0. \]

\[ f(x) \text{ is decreasing on an interval if and only if } f'(x) < 0. \]

Definitions

Another way to describe the graph of a function is to talk about concavity. Informally, $f(x)$ is **concave down** on an interval if it “bends down” on that interval, whereas $f(x)$ is concave up on an interval if it “bends up” on that interval.

It is important to know that on an interval, any combination of increasing/decreasing and concavity can occur.

Let’s do an example. Label the intervals where the graph is increasing, decreasing, concave up and concave down. 

**Definition:** A function has an inflection point at $x = a$ if the concavity $f(x)$ changes at $a$. 
Theory: Information from the First and Second derivative

As we said above:

1. \( f'(x) > 0 \) on an interval if and only if \( f(x) \) is increasing on that interval.
2. \( f'(x) < 0 \) on an interval if and only if \( f(x) \) is decreasing on that interval.
3. \( f'(x) = 0 \) on an interval if and only if \( f(x) \) is flat on that interval.

Now, suppose that \( f(x) \) is concave up on an interval, so that it is “bending up.” We can see that as \( x \) increases, the slope of the tangent line to the curve at \( x \) increases. This says that \( f'(x) \) is increasing. But according to the above statement, if \( f'(x) \) is increasing, then \( f''(x) > 0 \). Similarly if \( f(x) \) is concave down on an interval, \( f'(x) \) is decreasing and \( f''(x) < 0 \). So we have the following statements:

1. If \( f''(x) > 0 \) on an interval, then \( f(x) \) is concave up on that interval.
2. If \( f''(x) < 0 \) on an interval, then \( f(x) \) is concave down on that interval.

Examples

1. For the function \( f(x) = x^3 + 3/2x^2 - 18x + 1 \), find all the intervals on which the function is increasing, decreasing, concave up, and concave down.