Lecture Outline (Derivative of a Polynomial)  
Friday, February 1

Announcements

1. Quiz 3 on Monday 02/04/08 at 9:00.
2. Homework 3 is due on Wednesday 02/06/08.
3. Test grades will be posted on Coursework by Sunday. A summary of how the class did on the exam will be posted on my website.

Recap

Last time we talked a lot about notation and terminology associated with derivatives. We also briefly talked about taking “higher derivatives”.

Let’s do some examples:

Examples

1. Let $f(x) = x^3$. What are $f'(x)$ and $f''(x)$?
2. Let $y = \sqrt{x}$. What is $\frac{dy}{dx}$? What is $\frac{d^2y}{dx^2}$?

Notice that computing derivatives using the limit definition can get really nasty really fast.

The Power Rule

The Power Rule If $n$ is a positive integer, then
\[
\frac{d}{dx}(x^n) = nx^{n-1}.
\]

Why is this true?

It turns out that the power rule works for any real number $n$, not just when $n$ is a positive integer.

The (general) Power Rule If $n$ is any real number, then
\[
\frac{d}{dx}(x^n) = nx^{n-1}.
\]
Examples

1. If \( f(x) = x^8 \), find \( f'(x) \).
2. If \( y = x^{350} \), find \( y' \).
3. If \( y = t^5 \), find \( \frac{dy}{dt} \).
4. Find \( \frac{d}{dr}(\frac{1}{r^2}) \).
5. Differentiate \( y = \sqrt[3]{x^3} \).

Finding Derivatives from Old Ones

The constant multiple rule is:
If \( c \) is a constant and \( f \) is a differentiable function, then
\[
\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x)).
\]

We can check this using our limit definition of the derivative.
The Sum Rule is:
If \( f \) and \( g \) are both differentiable, then
\[
\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)).
\]

We can also check this using our limit definition of the derivative:

Now we can compute the derivative of any polynomial!!

Examples

1. \( \frac{d}{dx}(3x^4) \)
2. \( \frac{d}{dx}(x + 5x^2) \)