Lecture Outline
Wednesday, January 9

Theory: Interval Notation

Often we have to write down intervals, or combinations of intervals, on the real line. For example, we might write something like "the domain is all \( x \geq 0 \)." However, there is a more convenient notation that is usually used. This notation is best explained by example.

Note: Parentheses correspond to \(<\) and \(>\) and square brackets correspond to \(\leq\) and \(\geq\).

For multiple intervals, use the \( \bigcup \) symbol to combine intervals.

Examples: Interval Notation

1. Write \(-1 \leq x < 2\) or \(2 < x < 3\) or \(x \geq 4\) in interval notation.
2. What is the domain of \(y = \ln(x - 3)\)?
3. What is the domain of \(y = \frac{1}{x(x-2)}\)?

Theory: Piecewise Functions

A piecewise function is a function given by different formulas on different intervals. For example,

\[
f(x) = \begin{cases} 
  x^2 & : \ x < 0 \\
  x & : \ 0 \leq x < \pi \\
  \sin x & : \ x \geq \pi 
\end{cases}
\]

is a piecewise function that is equal to \(x^2\) when \(x\) is negative, equal to \(x\) when \(x\) is in the interval \([0, \pi)\), and equal to \(\sin x\) when \(x \geq \pi\).

Examples: Piecewise Functions

1. Graph the following piecewise function:

\[
f(x) = \begin{cases} 
  \ln x & : \ 0 < x < e \\
  1 & : \ e \leq x < 5 \\
  6 - x & : \ x \geq 5 
\end{cases}
\]

2. Be able to write down a piecewise function corresponding to a graph.
Theory: Function Composition

An important way of putting two functions together is **function composition**:

\[(f \circ g)(x) = f(g(x)).\]

To evaluate \(f \circ g\) at \(x\), first evaluate \(g\) at \(x\), and then plug this value into \(f\).